Unit 5 Lesson 4 Homework Set Solutions

Part I: More Matrix Operations

1. Determine whether the statement is true of false. If it is true, explain why it is true. If it is false, give an example to show why it is false. (Tan p. 123 #39-42).

a) If A and B are matrices of the same order and *c* is a scaler, then $c(A+B) = cA + cB$. **TRUE**

The matrix A+B is obtained by adding the corresponding entries of matrix A to those in matrix B. The matrix c(A+B) is then obtained by multiplying each element of the matrix A+B by the scalar c. The sum cA + cB is obtained by multiplying the matrix A by c and adding that to the matrix obtained by multiplying the matrix B by c.

b) If A and B are matrices of the same order, $A - B = A + (-1)B$. **TRUE**

The definition of subtraction is adding the opposite of a number, so A - B = A + (-1)B follows by matrix addition and the definition of subtraction.

c) If A is a matrix and *c* is a nonzero scalar, the $(cA)^T = (1/c)A^T$. **FALSE**

Suppose $A = \begin{pmatrix} 2 & 6 \\ -1 & 0 \end{pmatrix}$***, and c = 2. Then*** $(cA)^T = \begin{pmatrix} 4 & -2 \\ 12 & 0 \end{pmatrix}$***, but*** $(1/c)A^T = \begin{pmatrix} 1 & -1/2 \\ 3 & 0 \end{pmatrix}$.

d) If A is a matrix, then $(A^T)^T = A$. **TRUE**

A^T is obtained by interchanging the rows and columns of A. $(A^T)^T$ is obtained by interchanging the rows and columns of A^T. This gives us the original matrix A.

2. Let $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 4 & -2 \\ 2 & 1 \end{pmatrix}$ (Tan p. 133 #32).

a) Compute $(A+B)^2$:

$$(A+B)^2 = \begin{pmatrix} 47 & -10 \\ 20 & 7 \end{pmatrix}$$

b) Compute $A^2 + 2AB + B^2$.

$$A^2 + 2AB + B^2 = \begin{pmatrix} 49 & -15 \\ 18 & 5 \end{pmatrix}$$

c) From the results of parts (a) and (b), show that in general, $(A+B)^2 \neq A^2 + 2AB + B^2$.

Since $(A+B)^2 \neq A^2 + 2AB + B^2$ ***in parts a) and b), we know that, in general,*** $(A+B)^2 \neq A^2 + 2AB + B^2$.

[Problems from <u>Finite Mathematics</u>, by Soo Tan]

3. Let $A = \begin{pmatrix} 2 & 4 \\ 5 & -6 \end{pmatrix}$ $\quad B = \begin{pmatrix} 4 & 8 \\ -7 & 3 \end{pmatrix}$ (Tan p. 133 #33).

a) Find A^T and show that $(A^T)^T = A$: $\quad A^T = \begin{pmatrix} 2 & 5 \\ 4 & -6 \end{pmatrix} \quad (A^T)^T = \begin{pmatrix} 2 & 4 \\ 5 & -6 \end{pmatrix} = A$

b) Show that $(A+B)^T = A^T + B^T$:

$$(A+B)^T = \begin{pmatrix} 6 & -2 \\ 12 & -3 \end{pmatrix}$$

$$A^T + B^T = \begin{pmatrix} 6 & -2 \\ 12 & -3 \end{pmatrix}$$

c) Show that $(AB)^T = B^T A^T$:

$$(AB)^T = \begin{pmatrix} -20 & 62 \\ 28 & 22 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} -20 & 62 \\ 28 & 22 \end{pmatrix}$$

4. Cindy regularly makes long distance phone calls to three foreign cities—London, Tokyo, and Hong Kong. The matrices A and B give the lengths (in minutes) of her calls during peak and nonpeak hours, respectively, to each of these three cities during the month of June.

	London	Tokyo	Hong Kong
$A =$	80	60	40

	London	Tokyo	Hong Kong
$B =$	300	150	250

The cost for the calls (in dollars per minute) for the peak and nonpeak periods in the month in question are given, respectively, by the matrices

		C			D
C =	London	0.34	D =	London	0.24
	Tokyo	0.42		Tokyo	0.31
	Hong Kong	0.48		Hong Kong	0.35

Compute the matrix AC + BD, and explain what it represents. (Tan p. 138 #49).

$$AC = [71.6] \qquad BD = [206]$$

$$\Rightarrow AC + BD = [277.60]$$

So, Cindy's long distance bill for phone calls to those three cities is $277.60.

[Problems from <u>Finite Mathematics</u>, by Soo Tan]

Part II: Matrix Equations

5. The Great Farm has 500 acres of land allotted for cultivating corn and wheat. The cost of cultivating corn and wheat is $42 and $30 per acre, respectively. Mr. Great has $18,600 available for cultivating these crops. If he wants to use all the allotted land and his entire budget for cultivating these two crops, how many acres of each crop should he plant? (*Finite Mathematics*, Tan p. 93 #51)

$$42x + 30y = 18600 \qquad x + y = 500$$

$$\begin{bmatrix} 42 & 30 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18600 \\ 500 \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} 300 \\ 200 \end{bmatrix}$$

$$x = 300\,, \qquad y = 200$$

300 acres of corn and 200 acres of wheat should be cultivated.

6. The Coffee Cart sells a blend made with two different coffees, one costing $2.50 per pound, and the other costing $3.00 per pound. If the blended coffee sells for $2.80 per pound, how much of each coffee is used to obtain the blend? (Assume that the weight of the coffee bland is 100 pounds.) (*Finite Mathematics*, Tan p. 93 #53)

$$2.50x + 3.00y = 280 \qquad x + y = 100$$

$$\begin{bmatrix} 2.50 & 3.00 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 280 \\ 100 \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} 40 \\ 60 \end{bmatrix}$$

$$x = 40\,, \quad y = 60$$

40 lbs of Coffee 1 should be blended with 60 lbs of Coffee 2 to make the proper blend.

[Problems from Finite Mathematics, by Soo Tan]

7. The Math Movie Theater has a seating capacity of 900 and charges $2 for children, $3 for students, and $4 for adults. At a screening with full attendance last week, there were half as many adults as children and students combined. The receipts totaled $2800. How many adults attended the show? (*Finite Mathematics,* Tan p. 97 #60)

$$x + y + z = 2,800 \quad 2x + 3y + 4z = 900 \quad x + y - 2z = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 1 & -2 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2,800 \\ 900 \\ 0 \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} 200 \\ 400 \\ 300 \end{bmatrix}$$

$$x = 200\,, \quad y = 400\,, \quad z = 300$$

200 Children, 400 Students, and 300 adults attended.

8. The Toolees have a total of $100,000 to be invested in stocks, bonds, and a money market account. The stocks have a rate of return of 12% per year, while bonds pay 8% per year, and the money market account pays 4% per year. They have decided that the amount invested in stocks should be equal to the difference between the amount invested in bonds and 3 times the amount invested in the money market account. How should the Toolees allocate their resources if they require an annual income of $10,000 from their investments? (*Finite Mathematics,* Tan p. 106 #36)

$$x + y + z = 100,000 \quad .12x + .08y + .04z = 10,000 \quad x - y + 3z = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ .12 & .08 & .04 \\ 1 & -1 & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100,000 \\ 10,000 \\ 0 \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} 50,000 \\ 50,000 \\ 0 \end{bmatrix}$$

$$x = 50,000\,, \quad y = 50,000\,, \quad z = 0$$

$50,000 should be put into the stock market, $50,000 in bonds, and no investment should be made in a Money Market Account.

[Problems from Finite Mathematics, by Soo Tan]