Unit 5 Lesson 4 Homework Set

Part I: More Matrix Operations

1. Determine whether the statement is true or false. If it is true, explain why. If it is false, give an example to show why it is false. (Tan p. 123 #39-42).

      a. If A and B are matrices of the same dimension and c is a scaler, then $c(A+B) = cA + cB$.
      b. If A and B are matrices of the same dimension, $A - B = A + (-1)B$.
      c. If A is a matrix and c is a nonzero scalar, then $(cA)^T = (1/c)A^T$.
      d. If A is a matrix, then $(A^T)^T = A$.

2. Let $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 4 & -2 \\ 2 & 1 \end{pmatrix}$ (Tan p. 133 #32).

    a) Compute $(A+B)^2$.
    b) Compute $A^2 + 2AB + B^2$.
    c) From the results of parts (a) and (b), show that in general, $(A + B)^2 \neq A^2 + 2AB + B^2$.

3. Let $A = \begin{pmatrix} 2 & 4 \\ 5 & -6 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 8 \\ -7 & 3 \end{pmatrix}$ (Tan p. 133 #33).

    a) Find A^T and show that $(A^T)^T = A$.
    b) Show that $(A+B)^T = A^T + B^T$
    c) Show that $(AB)^T = B^T A^T$

4. Cindy regularly makes long distance phone calls to three foreign cities—London, Tokyo, and Hong Kong. The matrices A and B give the lengths (in minutes) of her calls during peak and non-peak hours, respectively, to each of these three cities during the month of June.

$$\begin{array}{ccc} \text{London} & \text{Tokyo} & \text{Hong Kong} \end{array}$$
$$A = \begin{pmatrix} 80 & 60 & 40 \end{pmatrix} \qquad \begin{array}{ccc} \text{London} & \text{Tokyo} & \text{Hong Kong} \end{array}$$
$$B = \begin{pmatrix} 300 & 150 & 250 \end{pmatrix}$$

The cost for the calls (in dollars per minute) for the peak and non-peak periods in the month in question are given, respectively, by the matrices

$$C = \begin{matrix} \text{London} \\ \text{Tokyo} \\ \text{Hong Kong} \end{matrix} \begin{pmatrix} 0.34 \\ 0.42 \\ 0.48 \end{pmatrix} \qquad D = \begin{matrix} \text{London} \\ \text{Tokyo} \\ \text{Hong Kong} \end{matrix} \begin{pmatrix} 0.24 \\ 0.31 \\ 0.35 \end{pmatrix}$$

Compute the matrix $AC + BD$, and explain what it represents. (Tan p. 138 #49).

Part II: Matrix Equations

5. The Great Farm has 500 acres of land allotted for cultivating corn and wheat. The cost of cultivating corn and wheat is $42 and $30 per acre, respectively. Mr. Great has $18,600 available for cultivating these crops. If he wants to use all the allotted land and his entire budget for cultivating these two crops, how many acres of each crop should he plant? (*Finite Mathematics,* Tan p. 93 #51)

6. The Coffee Cart sells a blend made with two different coffees, one costing $2.50 per pound, and the other costing $3.00 per pound. If the blended coffee sells for $2.80 per pound, how much of each coffee is used to obtain the blend? (Assume that the weight of the coffee bland is 100 pounds.) (*Finite Mathematics,* Tan p. 93 #53)

7. The Math Movie Theater has a seating capacity of 900 and charges $2 for children, $3 for students, and $4 for adults. At a screening with full attendance last week, there were half as many adults as children and students combined. The receipts totaled $2800. How many adults attended the show? (*Finite Mathematics,* Tan p. 97 #60)

8. The Toolees have a total of $100,000 to be invested in stocks, bonds, and a money market account. The stocks have a rate of return of 12% per year, while bonds pay 8% per year, and the money market account pays 4% per year. They have decided that the amount invested in stocks should be equal to the difference between the amount invested in bonds and 3 times the amount invested in the money market account. How should the Toolees allocate their resources if they require an annual income of $10,000 from their investments? (*Finite Mathematics,* Tan p. 106 #36)