Unit 5 Homework Set Solutions (Lesson 2 and L1-2 Additional Problems)

<u>Lesson 2 Set</u>

1. The following is a set of abstract matrices (without row and column labels):

$$M = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 2 & 4 & 1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \quad O = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \quad Q = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \quad R = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 0 & 2 \\ -1 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix} \quad U = \begin{bmatrix} 4 & 2 & 6 & -1 \\ 5 & 3 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{bmatrix}$$

List all orders of pairs of matrices from this set for which the product is defined. State the dimension of each product.

MO: 2x1, MP: 2x2, PM: 2x2, MR: 2x2, RM: 2x2, NQ: 3x1, NU: 3x4, PO: 2x1, US: 3x2, UT: 3x1…

2. The K.L. Mutton Company has investments in three states - North Carolina, North Dakota, and New Mexico. Its deposits in each state are divided among bonds, mortgages, and consumer loans. The amount of money (in millions of dollars) invested in each category on June 1 is displayed in the table below.

	NC	ND	NM
Bonds	13	25	22
Mort.	6	9	4
Loans	29	17	13

The current yields on these investments are 7.5% for bonds, 11.25% for mortgages, and 6% for consumer loans. Use matrix multiplication to find the total earnings for each state.

Total earnings for each state (in millions of dollars):

$$\begin{array}{ccc} \textit{Bonds} & \textit{Mort.} & \textit{Loans} \\ \end{array} \begin{bmatrix} 1.075 & 1.1125 & 1.06 \end{bmatrix} \begin{array}{c} \textit{Bonds} \\ \textit{Mort.} \\ \textit{Loans} \end{array} \begin{array}{c} \begin{array}{ccc} NC & ND & NM \end{array} \\ \begin{bmatrix} 13 & 25 & 22 \\ 6 & 9 & 4 \\ 29 & 17 & 13 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} NC & ND & NM \end{array} \\ \begin{bmatrix} 3.39 & 3.9075 & 2.88 \end{bmatrix} \end{array}$$

3. Several years ago Ms. Allen invested in growth stocks, which she hoped would increase in value over time. She bought 100 shares of stock A, 200 shares of stock B, and 150 shares of stock C. At the end of each year she records the value of each stock. The table below shows the price per share (in dollars) of stocks A, B, and C at the end of the years 1984, 1985, and 1986.

	1984	1985	1986
Stock A	68.00	72.00	75.00
Stock B	55.00	60.00	67.50
Stock C	82.50	84.00	87.00

Calculate the total value of Ms. Allen's stocks at the end of each year.

Total value of the stocks (in dollars) at the end of each year:

$$\begin{array}{ccc} A & B & C \\ \end{array} \begin{bmatrix} 100 & 200 & 150 \end{bmatrix} \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} 1984 & 1985 & 1986 \end{array} \\ \begin{bmatrix} 68 & 72 & 75 \\ 55 & 60 & 67.5 \\ 82.5 & 84 & 87 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} 1984 & 1985 & 1986 \end{array} \\ \begin{bmatrix} 30,175 & 31,800 & 34,050 \end{bmatrix} \end{array}$$

4. The Sound Company produces stereos. Their inventory includes four models - the Budget, the Economy, the Executive, and the President models. The Budget needs 50 transistors, 30 capacitors, 7 connectors, and 3 dials. The Economy model needs 65 transistors, 50 capacitors, 9 connectors, and 4 dials. The Executive model needs 85 transistors, 42 capacitors, 10 connectors, and 6 dials. The President model needs 85 transistors, 42 capacitors, 10 connectors, and 12 dials. The daily manufacturing goal in a normal quarter is 10 Budget, 12 Economy, 11 Executive, and 7 President stereos.

a. How many transistors are needed each day? Capacitors? Connectors? Dials?

b. During August and September, production is increased by 40%. How many Budget, Economy, Executive, and President models are produced daily during these months?

c. It takes 5 person-hours to produce the Budget model, 7 person-hours to produce the Economy model, 6 person-hours for the Executive model, and 7 person-hours for the President model. Determine the number of employees needed to maintain the normal production schedule, assuming everyone works an average of 7 hours each day. How many employees are needed in August and September?

Define the matrices for the inventory parts (I) and the daily manufacturing goal (N) as

$$I = \begin{array}{c} \\ B \\ Ec \\ Ex \\ P \end{array} \begin{array}{c} \begin{array}{cccc} t & ca & co & d \end{array} \\ \begin{bmatrix} 50 & 30 & 7 & 3 \\ 65 & 50 & 9 & 4 \\ 85 & 42 & 10 & 6 \\ 85 & 42 & 10 & 12 \end{bmatrix} \end{array} \quad and \quad N = \begin{array}{c} \begin{array}{cccc} B & Ec & Ex & P \end{array} \\ \begin{bmatrix} 10 & 12 & 11 & 7 \end{bmatrix} \end{array}$$

a. The answers are the results of the matrix multiplication

$$NI = \begin{array}{c} \begin{array}{cccc} t & ca & co & d \end{array} \\ \begin{bmatrix} 2810 & 1656 & 358 & 228 \end{bmatrix} \end{array}$$

b. The new daily manufacturing goals are given by

$$1.4N = \begin{array}{cccc} B & Ec & Ex & P \\ \end{array}$$
$$\begin{bmatrix} 14 & 16.8 & 15.4 & 9.8 \end{bmatrix}$$

Which should be rounded to integer quantities

c. Define a matrix H for hours of labor as

$$H = \begin{array}{c} \\ B \\ Ec \\ Ex \\ P \end{array} \begin{array}{c} Hrs. \\ \begin{bmatrix} 5 \\ 7 \\ 6 \\ 7 \end{bmatrix} \end{array}$$

The number of labor hours needed per week is given by

$$NH = 249$$

With 7-hour workdays, the number of employees needed is $\frac{249}{7} = 35.6$, which implies that 36 employees are needed to maintain full production. For August and September, we want $\frac{1.4NH}{7} = \frac{348.6}{7}$, which rounds to 50.

5. The president of the Lucrative Bank is hoping for a 21% increase in checking accounts, a 35% increase in savings accounts, and a 52% increase in market accounts. The current statistics on the number of accounts at each branch are as follows:

$$\begin{array}{r} \\ Northgate \\ Downtown \\ South\ Square \end{array} \begin{array}{c} \begin{array}{ccc} Checking & Savings & Market \end{array} \\ \begin{bmatrix} 40039 & 10135 & 512 \\ 15231 & 8751 & 105 \\ 25612 & 12187 & 97 \end{bmatrix} \end{array}$$

What is the goal for each branch in each type of account? (HINT: multiply by a 3×2 matrix with certain nonzero entries on the diagonal and zero entries elsewhere.) What will be the total number of accounts at each branch?

The goal for each branch in each type of account is given by:

$$\begin{array}{c} \\ N \\ S \\ D \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 630 & 420 & 250 \\ 650 & 350 & 275 \\ 700 & 370 & 150 \end{bmatrix} \end{array} \begin{array}{c} \\ c \\ s \\ m \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 1.21 & 0 & 0 \\ 0 & 1.35 & 0 \\ 0 & 0 & 1.52 \end{bmatrix} \end{array} = \begin{array}{c} \\ N \\ S \\ D \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 48448 & 13683 & 779 \\ 18430 & 11814 & 160 \\ 30991 & 16453 & 148 \end{bmatrix} \end{array}$$

Right-multiplying this result) by the matrix $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ yields the following total number of accounts at each branch: $\begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} Total \\ \begin{bmatrix} 62910 \\ 30404 \\ 47592 \end{bmatrix} \end{array}.$

6. On the Sunday before the 1986 NCAA basketball finals, a survey was taken of people's choices to win the game, along with their income. The following information was collected:

- 435 for Duke making over $30,000 per year
- 105 for Louisville making over $30,000 per year
- 115 with no choice making over $30,000 per year
- 125 for Duke making under $30,000 per year
- 205 for Louisville making under $30,000 per year
- 231 with no choice making under $30,000 per year

A survey was done in a bar in New York on the night of the game to determine the incomes of the people eating there (some were also watching the game). The following information was collected:

- 302 making over $30,000 per year
- 276 making under $30,000 per year

Using matrix operations, estimate the number of Duke fans, the number of Louisville fans, and the number of fans with no choice in the bar, based on the survey from Sunday. These numbers are found using probabilities based on the survey from Sunday and the data collected on the night of the game. Before you attempt to find an answer, the information from each survey should be converted to proportions and displayed in matrices.

The survey leads to the following matrix of probabilites:

$$\begin{array}{cc} & \begin{array}{ccc} \boldsymbol{Duke} & \boldsymbol{L'ville} & \boldsymbol{N.C.} \end{array} \\ \begin{array}{c} < 30000 \\ > 30000 \end{array} & \begin{bmatrix} \mathbf{125/561} & \mathbf{205/561} & \mathbf{231/561} \\ \mathbf{435/655} & \mathbf{105/655} & \mathbf{115/655} \end{bmatrix} \end{array}$$

We find the answer to our question by the following:

$$\boldsymbol{Number} \quad \begin{array}{c} \begin{array}{cc} < 30000 & > 30000 \end{array} \\ \begin{bmatrix} \mathbf{276} & \mathbf{302} \end{bmatrix} \end{array} \quad \begin{array}{cc} & \begin{array}{ccc} \boldsymbol{Duke} & \boldsymbol{L'ville} & \boldsymbol{N.C.} \end{array} \\ \begin{array}{c} < 30000 \\ > 30000 \end{array} & \begin{bmatrix} \mathbf{125/561} & \mathbf{205/561} & \mathbf{231/561} \\ \mathbf{435/655} & \mathbf{105/655} & \mathbf{115/655} \end{bmatrix} \end{array}$$

$$\approx \quad \begin{array}{c} \begin{array}{ccc} \boldsymbol{Duke} & \boldsymbol{L'ville} & \boldsymbol{N.C.} \end{array} \\ \begin{bmatrix} \mathbf{262} & \mathbf{149} & \mathbf{167} \end{bmatrix} \end{array}$$

7. A company that produces and markets stuffed animals has three plants – one on the East Coast, one on the West Coast, and one in the central part of the country. Among other items, each plant manufactures stuffed pandas, kangaroos, and rabbits. Personnel are needed to cut fabric, sew appropriate parts together, and provide finish work for each animal. Matrix A gives the time (in hours) of each type of labor required to make each type of stuffed animal; Matrix B gives the daily production capacity at each plant; Matrix C provides hourly wages of the different workers at each plant; and Matrix D contains the total orders received by the company in October and November.

$$A = \begin{array}{c} \\ Panda \\ Kangaroo \\ Rabbit \end{array} \begin{array}{c} \begin{array}{ccc} Cutting & Sewing & Finishing \end{array} \\ \begin{bmatrix} 0.5 & 0.8 & 0.6 \\ 0.8 & 1.0 & 0.4 \\ 0.4 & 0.5 & 0.5 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ East \\ Central \\ West \end{array} \begin{array}{c} \begin{array}{ccc} Panda & Kangaroo & Rabbit \end{array} \\ \begin{bmatrix} 25 & 15 & 12 \\ 10 & 20 & 15 \\ 20 & 15 & 15 \end{bmatrix} \end{array}$$

$$C = \begin{array}{c} \\ East \\ Central \\ West \end{array} \begin{array}{c} \begin{array}{ccc} Cutting & Sewing & Finishing \end{array} \\ \begin{bmatrix} 7.50 & 9.00 & 8.40 \\ 7.00 & 8.00 & 7.60 \\ 8.40 & 10.50 & 10.00 \end{bmatrix} \end{array}$$

$$D = \begin{array}{c} \\ Panda \\ Kangaroo \\ Rabbit \end{array} \begin{array}{c} \begin{array}{cc} Oct & Nov \end{array} \\ \begin{bmatrix} 1000 & 1100 \\ 600 & 850 \\ 800 & 725 \end{bmatrix} \end{array}$$

Use the matrices above to computer the following quantities:

We did these in class, so I will just list the matrices involved in the calculations.

a. The hours of each type of labor needed at each month (October, November) to fill all orders

$A^T D$ = …

b. The production cost per item at each plant

AC^T = …

c. The cost of filling all October orders at the East Coast plant

$D^T (AC^T)$ = …

d. The daily hours of each type of labor needed at each plant if production levels are at capacity

BA = …

e. The daily amount each plant will pay its personnel when producing at capacity

$(BA)C^T$ = …

Lessons 1-2 Additional Problems Set

1. Several years ago Ms. Allen invested in growth stocks, which she hoped would increase in value over time. She bought 100 shares of stock A, 200 shares of stock B, and 150 shares of stock C. At the end of each year she records the value of each stock. The table below shows the price per share (in dollars) of stocks A, B, and C at the end of the years 1984, 1985, and 1986.

	1984	1985	1986
Stock A	68.00	72.00	75.00
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Stock C	82.50	84.00	87.00

Calculate the total value of Ms. Allen's stocks at the end of each year.

Same as #3 in Lesson 2 Set.

2. A virus hits campus. Nurse Nancy discovers that students are either sick, well, or carriers of the virus. She finds the following percentages of people in each category, depending on whether they are a junior or a senior.

The student population is distributed by class and sex as follows:

	Junior	Senior
Well	15%	25%
Sick	35%	40%
Carrier	50%	35%

	Males	Females
Junior	104	80
Senior	107	103

How many sick males are there? How many well females? How many female carriers?

Sick males = 79, well females = 38, and females carriers = 76. These numbers are derived from the following matrix multiplication:

$$\begin{array}{c} \\ \mathbf{Well} \\ \mathbf{Sick} \\ \mathbf{Car.} \end{array} \begin{array}{c} \quad \mathbf{Jr.} \quad \mathbf{Sr.} \\ \begin{bmatrix} .15 & .25 \\ .35 & .40 \\ .50 & .35 \end{bmatrix} \end{array} \begin{array}{c} \\ \mathbf{Jr.} \\ \mathbf{Sr.} \end{array} \begin{array}{c} \quad \mathbf{m} \quad \mathbf{f} \\ \begin{bmatrix} 104 & 80 \\ 107 & 103 \end{bmatrix} \end{array} = \begin{array}{c} \\ \mathbf{Well} \\ \mathbf{Sick} \\ \mathbf{Car.} \end{array} \begin{array}{c} \quad \mathbf{m} \quad \mathbf{f} \\ \begin{bmatrix} 42.35 & 37.75 \\ 79.2 & 69.2 \\ 89.45 & 76.05 \end{bmatrix} \end{array}$$

3. The president of the Lucrative Bank is hoping for a 21% increase in checking accounts, a 35% increase in savings accounts, and a 52% increase in market accounts. The current statistics on the number of accounts at each branch are as follows:

$$\begin{array}{cc} & \begin{array}{ccc} \textit{Checking} & \textit{Savings} & \textit{Market} \end{array} \\ \begin{array}{r} \textit{Northgate} \\ \textit{Downtown} \\ \textit{South Square} \end{array} & \begin{bmatrix} 40039 & 10135 & 512 \\ 15231 & 8751 & 105 \\ 25612 & 12187 & 97 \end{bmatrix} \end{array}$$

What is the goal for each branch in each type of account? (HINT: multiply by a 3×2 matrix with certain nonzero entries on the diagonal and zero entries elsewhere.) What will be the total number of accounts at each branch?

Same as #5 in Lesson 2 Set

4. Winners at a science fair are determined by a scoring system based on five items with different weights attached to each item. The items and associated weights are the summary of background research – weight 3; experimental procedure – weight 5; research paper – weight 6; project display – weight 8; and creativity of idea – weight 4. Each project is judged by grading each of the five items on a scale from 0 to 10, with 10 highest. The total score for a project is derived by adding the products of the corresponding weights and points for each item.

	Peter	Jane	Bryan	Kathy	Mary	Chris	John
Background research	9	8	10	7	8	9	10
Experimental procedure	10	9	9	10	10	9	10
Research paper	7	9	8	9	7	8	8
Project display	9	10	9	8	10	8	9
Creativity of idea	8	7	8	10	6	8	7

a. What is the maximum total score possible for a project?

$$\mathbf{10(3+5+6+8+4)=260}$$

b. Calculate the score for a student who earns 8 points on background research, 9 points on experimental procedure, 7 points on the research paper, 8 points on the project display, and 6 points on creativity.

$$
\begin{array}{cccccc}
 & BR & EP & RP & PD & CI \\
\textit{Weight} & [3 & 5 & 6 & 8 & 4]
\end{array}
\begin{array}{c}
 \\ BR \\ EP \\ RP \\ PD \\ CI
\end{array}
\begin{array}{c}
\textit{Score} \\
\begin{bmatrix} 8 \\ 9 \\ 7 \\ 8 \\ 6 \end{bmatrix}
\end{array}
= [199]
$$

c. The table shown below contains the points for the finalists in the biology division. Calculate the total scores to determine the first, second, and third place entries.

Extend part b) with the following operation:

$$
\begin{array}{cccccc}
 & BR & EP & RP & PD & CI \\
\textit{Weight} & [3 & 5 & 6 & 8 & 4]
\end{array}
\begin{array}{c}
 \\ BR \\ EP \\ RP \\ PD \\ CI
\end{array}
\begin{array}{c}
P \quad Ja \quad B \quad K \quad M \quad C \quad Jo \\
\begin{bmatrix}
9 & 8 & 7 & 8 & 9 & 10 \\
10 & 9 & 9 & 10 & 9 & 10 \\
7 & 9 & 8 & 7 & 8 & 8 \\
9 & 10 & 9 & 10 & 8 & 9 \\
8 & 7 & 8 & 6 & 8 & 7
\end{bmatrix}
\end{array}
$$

$$
= \begin{array}{ccccccc}
P & Ja & B & K & M & C & Jo \\
[223 & 231 & 227 & 229 & 220 & 216 & 228]
\end{array}
$$