Applied Finite Math
Trees Graded Assignment Solutions

50. a. $2N - 2$: A tree with N vertices has $N - 1$ edges, and, in any graph, the sum of the degrees of all the vertices is twice the number of edges.

b. Let v be the number of vertices in the graph, e the number of edges, and k the number of vertices of degree 1. Recall that in a tree $v = e + 1$ and in any graph the sum of the degrees of all the vertices is $2e$. Now, since we are assuming there are exactly k vertices of degree 1, the remaining $v - k$ vertices must have degree at least 2. There for the sum of the degrees of all the vertices must be at least $k + 2(v - k)$. Putting all this together we have $2e \geq k + 2(v - k) = k + 2(e + 1 - k), 2e \geq k + 2e + 2 - 2k, k \geq 2.$

52. a. If $R = 0$, then $M = N - 1$ (the number of edges is one less that the number of vertices). So, the network is a tree.

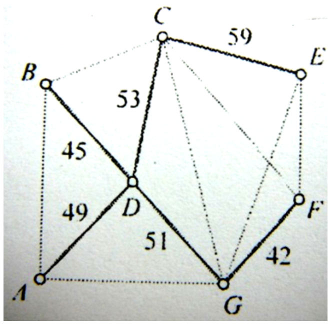
b. If $R = 1$, then $M = N$ (the number of edges is the same as the number of vertices). So, the network is not a tree. From the definition of a tree, there must be at least one circuit. Suppose the graph had 2 (or more) circuits. Then, there would be an edge of one of the circuits that is not an edge of another circuit. Removing such an edge would leave us with a connected graph with the number of vertices being one more that the number of edges (i.e. a tree). But this tree would have a circuit. This is impossible, so there cannot be more than 1 circuit.

c. The maximum redundancy occurs when the degree of each vertex is $N - 1$. In that case (a complete graph), the number of edges is $(N - 1) + (N - 2) + \cdots + 3 + 2 + 1$. So $R = (N - 1) + (N - 2) + \cdots + 3 + 2 + 1 - (N - 1) = (N - 2) + \cdots + 3 + 2 + 1 = (N^2 - 3N + 2)/2.$

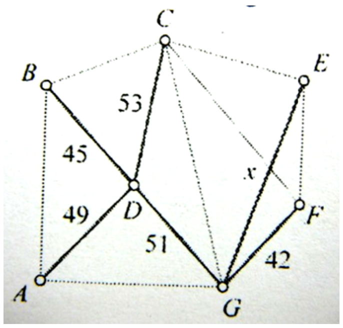
54. The minimum spanning tree consists of the following edges (produced exactly in this order by Kruskal's algorithm): C_4C_7, C_3C_6, C_1C_8, C_1C_2, C_2C_5, C_2C_6, C_8C_9, and C_7C_9. (The total cost is 15 million dollars).

56.

a. The MST is as the same as given in Figure 7-67(a). If x > 59, then edge *EG* will not replace any edge in the existing MST since it has a larger value.

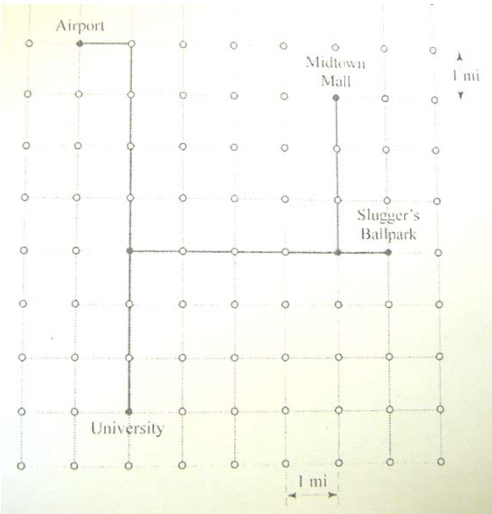


b. If x < 59, then edge *EG* will replace edge *CE* in the MST.



58.

a. The switching stations could be located 3 blocks north of the University and 1 block west of Slugger's Ballpark as shown. The answer, however, is not unique. For example, the switching station could be moved 1 or 2 blocks north from the location shown.



b. Since the optimal network requires 16 miles of track and 2 switching stations, the cost of the network is $33 million.