Curriculum of the Algebra 2 Collaboration
NCSSM Distance Learning and North Carolina Schools
2001-2002

Linear Data Analysis

1. **Guess the Age**
   Students are asked to guess the ages of a group of famous people. The actual age is paired with the student's guess to begin discussion of information in a scatterplot, a linear model ($y=x$), and the accuracy of the model. In an attempt to identify the best guesser in the class, techniques are used that anticipate residuals.
   Technology: PowerPoint slideshow to show faces, Graphing calculator

2. **Introduction to Linear Data Analysis**
   Using a collection of eight scatterplots, students determine graphs of lines or curves that might be appropriate models to describe bivariate data. Data that associates the length of a spring with the weight hung from the spring is used to find the linear regression line. Interpret the slope and the *y*-intercept of the line, and forecast other ordered pairs from the linear model.
   Technology: PowerPoint slideshow of scatter plots, Graphing calculator

3. **Price of Apples**
   More practice with scatterplots, linear regression line, interpreting slope and *y*-intercept, and prediction. Students will shift data left to give meaning to the *y*-intercept. Students discuss possible criteria of linear regression line. (Taken from the Algebra II Indicators from NCDPI.)
   Technology: Graphing calculator

4. **Hurricane Fran**
   The criteria of the linear regression line are defined. The Geometer's Sketchpad geometric illustration is useful to showing the sum of the squares of the residuals. The Fran data is definitely not linear. Fitting this data with a line will show the usefulness of the residual plot. (Taken from the Algebra II Indicators from NCDPI.)
   Technology: Geometer's Sketchpad web site, The WRAL website, Graphing calculator.

5. **Piecewise Defined Functions as Models**
   The data for the Olympic swimming records for the 400 Freestyle found in the Algebra 2 Indicators show data with two definite trends over time. In this lesson students develop a piecewise-defined linear function using domain restrictions and the linear regression line. This model provides specific information in the slopes to compare the data of the two trends. Since both men's and women's data is given, one data set can be discussed in class and students can follow up the lesson with the other data set.
   Technology: Graphing Calculator

The Linear Function

6. **Linear Inequalities with a Parallelogram**
   Students are given the coordinate of a point in the second quadrant. Based on this point they develop a group of four linear inequalities whose solution forms a parallelogram that falls in the second quadrant of the coordinate system. The task asks students to find equations of lines and look at points of intersection. They learn to graph inequalities on the graphing calculator. (Taken from the Algebra II Indicators from NCDPI.)
   Technology: Geometer's Sketchpad and Graphing calculator.

7. Linear Programming

Using a problem setting of varying hours of summer work with two possible jobs, students explore the restrictions of domain and range and linear inequalities to set boundaries on a region of the graph where solutions may lie. Within this region, they must determine the solution that gives the maximum weekly income. (This problem is based on a problem from the NC Algebra II Indicators.) Once a solution is reached, generalizations about the method are made, and another problem tackled.
Technology: Graphing calculator, PowerPoint slideshow.

<u>The Quadratic Function</u>

8. Wile E. Coyote

Wile E. Coyote creates a catapult to catch Road Runner. Using a quadratic function that describes the trajectory of the Wile E. as he is shot from the catapult. Students find maximum values, zeros, and domain to answer questions about the antics of Wile E. (This problem is based on a talk by Wally Dodge.) A second problem from the NC Algebra II Indicators leads students through a similar procedure in tracking the path of a space shuttle.
Technology: Graphing calculator, Animation (from studio video) showing Wile E. and Road Runner.

9. Pig Problem: Writing and Solving Quadratic Equations

Given several problem settings, students develop quadratic functions for which they investigate maximum values, zeros, and specific values to answer specific questions about the settings.
Technology: Flash animations to illustrate problem settings, Graphing calculator.

10. Football and Braking Distance: Model Data with Quadratic Functions

Students are given data to describe the trajectory of a football tossed from the tallest bleachers of a stadium. The data is fit with a quadratic function using least squares criteria. Given data extracted from page 288 of Glencoe's Algebra II book, students investigate braking distance versus speed of a car. Using quadratic least squares, the student finds a best-fit function for the data. Data is given on reaction distance versus speed of the car. When reaction distance is added to braking distance to find total stopping distance, students fit another quadratic function. A Follow Up Problem relates number of sides of a polygon with the number of vertices to create a quadratic function.
Technology: Flash animation to illustrate problem setting, Graphing calculator.

11. Questions about Quadratics

Using a group of questions from the Algebra II Indicators from DPI, students use both the calculator and paper and pencil to answer questions about characteristics of quadratic functions.
Technology: Graphing calculator.

12. **Collecting and Fitting Quadratic Data with the CBL**

Using the CBL and the graphing calculator, students work in groups to collect data describing the freefall of an object over time. The data collected includes data not relevant and that must be eliminated, and data is shifted near the y-axis to make the intercept meaningful. The students describe the meaning of the coefficients. The experiment is run again with an object that has drag (like a hat) and a model is found. The follow-up problem works with the football data from the lesson: Football and Braking Distance: Model Data with Quadratic Functions.
Technology: CBL, Graphing calculator, Balldrop and Hiker programs for the TI-83 plus.

Other Functions

13. Distance Formula

Using rulers, students measure distances on a diagram to find a shortest path. They create ordered pairs and a scatterplot. With the motivation that the scatterplot has a clear message, the students develop a function that measures the distances using the distance formula. Based on the function, the shortest distance can be estimated and then considered on the diagram. A follow-up problem involving determining the best place to put a new Post Office is included.
Technology: Ruler, Graphing Calculator

14. Equations with Radical Expressions

Data representing the period of a swinging pendulum versus the length of the pendulum can be best modeled by a square root function. Data and an appropriate model are both given to the students. Questions from the NC Algebra II Indicators require students to solve equations involving radical expressions. Solutions are also investigated from both a graphical and an analytical point of view.
Technology: Graphing Calculator

15. Applications of Rational Functions

By developing a function to describe the annual cost of a refrigerator and given a function describing concentration of drug in the body, students relate the behavior of the graph of a rational function with the phenomenon it describes. Asymptotes and particular points become important information about the application.
Technology: Graphing calculator.

16. Composition and Inverses of Functions

Concepts of composition are used to develop functions that describe volumes of pyramids with specific bases and combinations of special discounts when purchasing a car. The connection between study time and number of courses leads to a function using inverse function that can help students determine the number of courses to take for available weekly study time.
Technology: Graphing calculator.

17. Polynomials as Models

A data set of the average price of gasoline for each year from 1993 to 2001 shows data with many changes. Using all the different regression curves and the regression line from the calculator, the students investigate the best model of the data and discuss its ability to predict.
Technology: Graphing calculator

The Exponential Function

18. The Drug Problem

Using ideas presented in Jim Sandefur's article from the February 1992 *Mathematics Teacher* we model the amount of cough syrup in the body over time with water and food coloring. Next, students calculate the amounts of medicine in the body every four hours using an informal iterative process. From these ordered pairs of time and amount, we fit the data with an exponential function found using the exponential regression fit on the calculator. This function is then interpreted within the context of the amount of cough syrup in the body.
Technology: Measuring cups, food coloring and spoons, Graphing calculator.

19. Half-life and Doubling Time

Skittles or M&M's are randomly thrown onto a paper plate. The candies that fall with a letter face up are removed. Students document throw number and number of pieces remaining. Using the exponential regression fit, we find a decreasing exponential function with a half-life of one. A similar data collection that leads to an increasing exponential function with a doubling time of one results from cutting a sheet of paper, stacking the resulting pieces and cutting again. With these definitions, the Hurricane Fran data from the Algebra 2 indicators is fit with an exponential function. Students then determine if this data has a half-life or doubling time.

Technology: M&M's, scissors, Graphing calculator.

20. Money and the Exponential Function

Using the ideas of compounding, students use shorter and shorter compounding periods that lead to the definition and meaning of *e*.

Technology: Graphing calculator.

21. Voltage Data Collection for Exponentials

Using the CBL with voltage probe, a 9-volt battery, resistor, and capacitor, students collect data describing how the voltage drains from a capacitor when it is disconnected from the battery. A comparison of the ratios of the voltage reading at one second with the voltage reading of the next second reveals that the voltage is falling by a consistent percentage. Therefore, the data is described by a decaying exponential function as a model.

Technology: Graphing calculator, CBL with voltage probe, battery, resistor, and capacitor for each group of students.

<u>Mathematical Modeling</u>

22. The Box Problem

Students build open top rectangular boxes from a standard sheet of paper by cutting congruent squares from each corner. Data is collected that pairs the length of the side of the cut out square with the volume of the resulting box. To describe a clear pattern shown in the scatter plot, students develop a function through analysis of the box design. Based on this function, the length of the side of the square is determined that will create a box of maximum volume, and two squares that will produce a box of equal volume.

Technology: Graphing calculator, Geometer's Sketchpad sketch, Animation (from studio video) showing 3 different versions of how to make a box.

23. Relationships in Rectangles

Using random integers between 0 and 30, students create lengths and widths of rectangles. In the list facility of the graphing calculator, these lengths and widths can be used to calculate perimeters and areas of the rectangles. Students investigate several relationships using scatter plots—the most exciting is area versus perimeter. Using the ideas of a function forming a boundary on the scatter plot, students discover information about the perimeter and area of a rectangle. This lesson is based on an article "Connecting Data and Geometry" by Tim Erickson found in the November 2001 *Mathematics Teacher*.

Technology: Graphing calculator.

<u>Miscellaneous Topics</u>

24. Univariate Data Analysis

Using the techniques of line plots and stem and leaf plots, but focused on box and whisker plots, students investigate which baseball player they would most like to have on their team: Barry Bonds, Mark McGwire, or Sammy Sosa.

Technology: Graphing calculator, data program for TI-83 plus.

25. Matrix Operations

Using three settings students apply matrix addition and multiplication. Statistics of recent NFL quarterbacks from several years allow students to see the definition of matrix addition, matrix subtraction, and scalar multiplication. Using a problem setting from *Contemporary Precalculus through Applications* students investigate orders of students at a lunch counter using a probability, the transition matrix, and matrix multiplication. A similar technique is used to investigate the location of a mouse in a maze as a follow-up activity

Technology: PowerPoint slideshow for NFL problem, Flash animation for mouse in the maze problem, graphing calculator.

26. Complex Numbers

Using the definition of complex numbers and operations with complex numbers, students add, multiply, and graph with complex numbers using some sample items from the NC Algebra II Indicators. Once familiar with the operations and graphing, students iterate complex numbers in functions to determine whether the iteration stabilizes. With some experimentation, rules are developed that show patterns in stabilization that carry into graphs by special coloring schemes. The result is a fractal. Examples from the Julia Set and the Mandelbrot Set are shown.

Technology: Graphing calculator, Power Point slides.

27. End of Course Test

Using nine sample items from the Algebra 2 End of Course test, students develop strategies for taking the end of course test.

Technology: Graphing calculator.