Sample Items from the EOC
Algebra 2

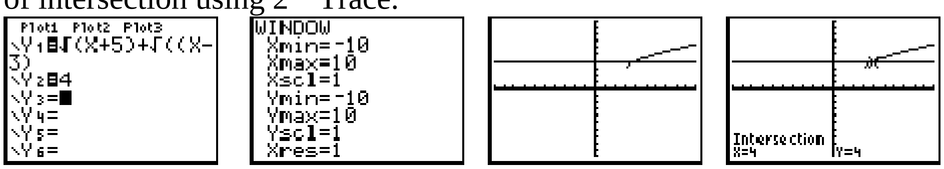
Goals:
1. Investigate strategies in answering items that are multiple choice.
2. Compare analytical, graphical, and technology-based methods of solving problems.

Materials and Equipment Needed:
1. Copy of student handout
2. Graphing calculator.

Activity: Discussing and answering the questions.
Sample Items were taken from
www.ncpublicschools.org/accountability/testing/eoc/algebra2/sampleitems.asp.
Students should try the question for themselves and then be given time to discuss ways they might solve the question. Hopefully, a number of ways of thinking about the problem will be presented. The teacher notes contain some of these methods, but have no guarantees of being complete.

Question 1:
1. Solve $\sqrt{x+5}+\sqrt{x-3}=4$
   a) $\{4\}$
   b) $\left\{\frac{1}{4},1\right\}$
   c) $\{-1,4\}$
   d) no solution

- Using the graphing calculator, let $Y1=\sqrt{x+5}+\sqrt{x-3}$ and $Y2=4$ and find the point of intersection using 2nd Trace.



  One caution when using the intersection option, the cursor on the graph must be on a defined ordered pair of the curve. For $Y1=\sqrt{x+5}+\sqrt{x-3}$ move the cursor into region where $x>3$ using the right arrow key.
- This problem can be analytically by isolating the radicals, squaring both sides of the equation to remove radicals, and then that procedure again.

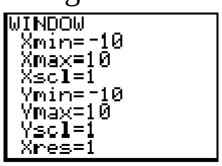
$$\sqrt{x+5}+\sqrt{x-3}=4$$
$$\sqrt{x+5}=4-\sqrt{x-3}$$
$$x+5=16-8\sqrt{x-3}+x-3$$
$$8\sqrt{x-3}=8$$
$$x=4$$

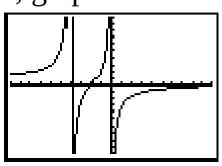
- Since this item is multiple choice, essentially the answers given can be tested. Given that the question includes the expression $\sqrt{x-3}$, the values must be $x \geq 3$. This information eliminates choices b and c. By testing $x=4$ in $\sqrt{x+5}+\sqrt{x-3}=4$, we find this works. Therefore the answer is a.

2. Which set contains the zeros of $f(x)=\dfrac{x}{x+4}-\dfrac{3}{x}-\dfrac{1}{2}$?

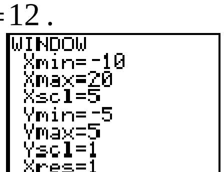
   a. $\{-6,-4\}$
   b. $\{-12,2\}$
   c. $\{6,4\}$
   d. $\{12,-2\}$

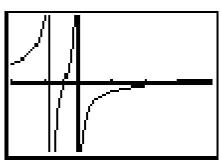
- Using the calculator, graph the function to find the zeros (where $y=0$).





Looking at the answers that are possible we should see that the window must show $x=12$.





Using the 2^{nd} Trace zeros are found at $\{12,-2\}$.

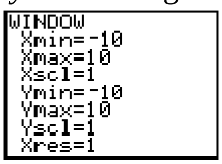
- Using analytical methods, the same result is seen.

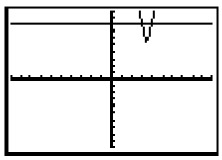
$$y=\frac{x}{x+4}-\frac{3}{x}-\frac{1}{2}$$
$$=\frac{2x(x)-3(x+4)(2)-x(x+4)}{2x(x+4)}$$
$$=\frac{x^2-10x-24}{2x(x+4)}$$
$$=\frac{(x-12)(x+2)}{2x(x+4)}$$

- Since the problem is multiple choice, answer can be substituted. This can be done efficiently on the calculator by defining the function as Y1 and finding $Y1(-6)$ on the home screen.

3. For $y = 3|7-2x|+5$, which set describes x when $y < 8$?
   a. $\{x|3 < x < 4\}$
   b. $\{x|3 < x < 10\}$
   c. $\{x|x < 3 \ or \ x > 4\}$
   d. $\{x|x < 3 \text{ or } x > 10\}$

- Graph both $Y1 = 3|7-2x|+5$ and $Y2 = 8$ to determine where Y1 falls below the line $y = 8$. Using the standard window:





Looking at the possible answers, the only option for the answer is a. You can use 2nd Trace to find points of intersection or the table if needed.
- Solving this analytically
$3|7-2x|+5 < 8$

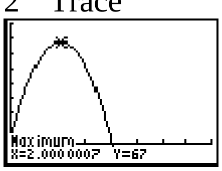
$|7-2x| < 1$

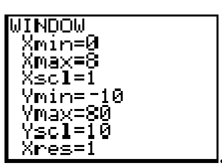
$-1 < 7-2x < 1$

$3 < x < 4$
- Students could use the four choices to select x-values to test in the $y = 3|7-2x|+5$ to determine which produce values less than 8.

4. A ball is thrown upward. Its height (h in feet) is given by the function $h = -16t^2 + 64t + 3$ where t is the length of time (in seconds) that the ball has been in the air. What is the maximum height that the ball reaches?
   a. 3 feet
   b. 51 feet
   c. 63 feet
   d. 67 feet

- Using the calculator, graph the function and find the maximum point at (2, 67) with 2nd Trace





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- To solve this analytically, complete the square of this quadratic to determine the coordinates of the vertex. This will produce the maximum point.

$$h = -16t^2 + 64t + 3$$
$$= -16(t^2 - 4t \qquad) + 3$$
$$= -16(t^2 - 4t + 4) + 3 + (16 \cdot 4)$$
$$= -16(t-2)^2 + 67$$

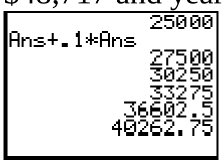
Thus the vertex is located at (2,67) which gives 67 as the maximum value.

- Working from the choices does not give any easy methods for finding the maximum.

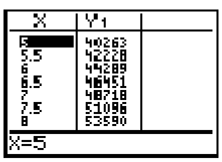
5. Mr. Jones bought a piece of property for $25,000. If the property appreciates at a rate of 10% per year, what will be its ***approximate*** value in $7\frac{1}{2}$ years?

   a. $53,000
   b. $51,000
   c. $44,000
   d. $39,000

- Using the home screen and the formula $ANS + 0.1 \cdot ANS$ with the initial value of 25000, does not give good results because of needing 7.5 years. Year 7 produces $48,717 and year 8 produces $53,589.



However, using a table of values for $Y1 = 25000\left(1 + \frac{0.1}{1}\right)^{1 \cdot x}$ will show



which identifies b as the correct answer. One word of caution: do not use the compounding continuously formula $y = 25000e^{0.1x}$ in this problem. The value for $x = 7.5$ is $52925 which leads you to answer a—which is not correct.

- This is not really feasible to do analytically.
- Working from the answers backward does not help in this problem.

6. If $f(x) = x^2 - x$ and $g(x) = x - 1$, what is $f(g(x))$?

   a. $x^2 - x - 1$
   b. $x^2 - x - 2$
   c. $x^2 - 3x + 2$
   d. $x^2 - 3x + 1$

- The calculator does not really help on this problem.
- The analytical solution requires that you know the definition of composition of functions.

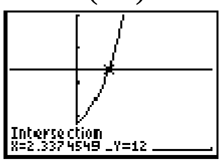
$$f(g(x)) = f(x-1)$$
$$= (x-1)^2 - (x-1)$$
$$= x^2 - 3x + 2$$

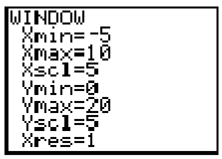
- Working backwards, we know that $f(0) = 0$ and $g(1) = 0$. Therefore $f(g(1)) = 0$. Looking at all the answers, the only expression that gives 0 where $x = 1$ is c.

7. If $y = 4(1.6)^x$, what is the ***approximate*** value of x when $y = 12$.

   a. 2.5
   b. 2.3
   c. 2.1
   d. 1.9

- Using the graphing calculator, find the point of intersection (using 2nd Trace) between $Y1 = 4(1.6)^x$ and $Y2 = 12$.





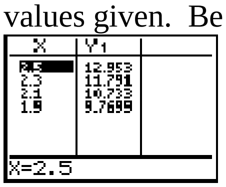
This gives us b as an answer.

- Doing this problem analytically requires the use of logarithms.

$$4(1.6)^x = 12$$
$$(1.6)^x = 3$$
$$\log(1.6^x) = \log(3)$$
$$x \cdot \log(1.6) = \log(3)$$
$$x = \frac{\log(1.6)}{\log(3)}$$

This also gives 2.3 as the answer.

- Working from the answers, substitute into the equation $Y1 = 4(1.6)^x$ for each of the values given. Be sure to use approximate answers.



8. The load that a beam with constant length can support varies jointly with its width and the square of its height. If a beam 12 feet long, 1 foot wide, and 3 inches high can support a load of 62.5 pounds, how much weight can be supported by a beam 12 feet long, 2 feet wide, and 6 inches high?

   a. 125 pounds
   b. 250 pounds
   c. 500 pounds
   d. 1,000 pounds

- Graphing is not useful in this problem. There are too many varibles.

- An analytical solution begins with writing an equation that describe the relationship through variation.

  $load = k \cdot weight \cdot (height)^2$

  Substitute values from the beam for which the load is given. Notice length must be constant which is true of the two beams.

  $62.5 = k \cdot 1 \cdot (3)^2$

  $6.944 = k$

  Using this value of k in the formula for the next beam

  $load = 6.944 \cdot 2 \cdot (6)^2$

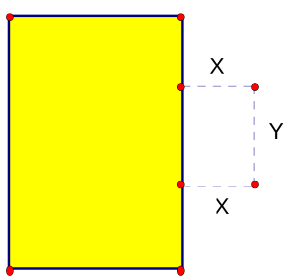
  $load = 500$

  Thus c is the correct answer.

9. The director of a local preschool plans to enclose a rectangular area for a playground. One side will be the side of the building itself. If 60 feet of fence are to be used, what is the ***maximum*** area that can be enclosed?

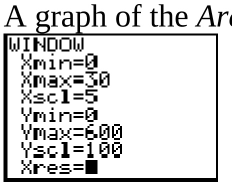
   a. 575 ft^2
   b. 450 ft^2
   c. 400 ft^2
   d. 225 ft^2

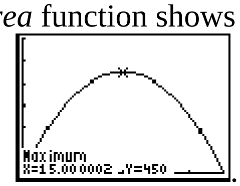
- To begin any method, a figure is needed.

  We know that $2x + y = 60$ and $Area = xy$. Solving for y in the first equation produces $y = 60 - 2x$. Therefore $Area = x(60 - 2x)$.

  A graph of the *Area* function shows a maximum.

  . The answer is b.

- To solve this analytically, complete the square on $Area = x(60 - 2x)$. However, since the zeros can be seen by inspection at $x = 0$ and $x = 30$, the maximum must be exactly between these zeros at $x = 15$. Let $x = 15$ which produces $Area = 450$.
- There is no effective way to work backwards from these answer.

Answers: A, D, A, D, B, C, B, C, B

Student Handout
Sample Items for the NC End-of-Course Test
Algebra 2

Taken from www.ncpublicschools.org/accountability/testing/eoc/algebra2/sampleitems.asp

The following items were developed for the NC EOC Test for Algebra II and are aligned with the 1998 Algebra II Curriculum.

Select the best answer:

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   a. $\{4\}$
   b. $\left\{\frac{1}{4},1\right\}$
   c. $\{-1,4\}$
   d. no solution

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