Algebra 2
Matrix Operations

Goals:

- Translate among graphic, algebraic, and verbal representations of relations. (3.02)
- Operate with matrices to solve problems
  a. Add, subtract, and multiply matrices (4.04)

Materials Needed:

- Graphing calculators
- Copy of student handout
- There is no follow up handout. The mouse problem may be used as a follow up question.

Activity One: Investigate Football Statistics using Operations with Matrices

The idea for this problem comes from page 13 of Contemporary Mathematics in Context, by Coxhead et al., copyright 1998.

1. The tables below represent statistics on several NFL quarterbacks for 1998 and 1999 seasons. These statistics come from http://sportsillustrated.cnn.com/football/nfl/rosters/Quarterbacks.html. Use the powerpoint presentation: matricesfootball.ppt

1998 Statistics				
Player	Attempts	Completions	Touchdowns	Interceptions
Troy Aikman	315	187	12	5
Tony Banks	408	241	7	14
Jeff Blake	93	51	3	3
Steve Beuerlein	343	216	17	12

1999 Statistics				
Player	Attempts	Completions	Touchdowns	Interceptions
Troy Aikman	442	263	17	12
Tony Banks	320	169	17	8
Jeff Blake	389	215	16	12
Steve Beuerlein	571	343	36	15

As matrices they are $B = \begin{bmatrix} 315 & 187 & 12 & 5 \\ 408 & 241 & 7 & 14 \\ 93 & 51 & 3 & 3 \\ 343 & 216 & 17 & 12 \end{bmatrix}$ and $A = \begin{bmatrix} 442 & 263 & 17 & 12 \\ 320 & 169 & 17 & 8 \\ 389 & 215 & 16 & 12 \\ 571 & 343 & 36 & 15 \end{bmatrix}$

2. If the 1999 statistics are named matrix A and the 1998 statistics are named matrix B, discuss the meaning of $A-B$, $A+B$, , $\frac{1}{2}A$.

$$A - B = \begin{bmatrix} 127 & 76 & 5 & 7 \\ -88 & -72 & 10 & -6 \\ 296 & 164 & 13 & 9 \\ 228 & 127 & 19 & 3 \end{bmatrix} \quad A + B = \begin{bmatrix} 757 & 450 & 29 & 17 \\ 728 & 410 & 24 & 22 \\ 482 & 266 & 19 & 15 \\ 914 & 559 & 53 & 27 \end{bmatrix}$$

$$\frac{1}{2}A = \begin{bmatrix} 221 & 131.5 & 8.5 & 6 \\ 160 & 84.5 & 8.5 & 4 \\ 194.5 & 107.5 & 8 & 6 \\ 285.5 & 171.5 & 18 & 7.5 \end{bmatrix}$$

$A - B$ gives the difference in stats from 1998 to 1999. $A + B$ gives the total stats for two years. $\frac{1}{2}A$ gives an estimate of standings midseason in 1999.

Activity Two: Multiplication of Matrices in Buying Patterns

This problem comes from page 537 of <u>ContemporaryPrecalculus through Applications</u>*, by Barrett et al., copyright 2000.*

1. The school cafeteria sells three items that students especially like: onion rings, French fries, and chocolate chip cookies. The manager noticed that each student ordered depending on what he or she ordered on the previous visit. A survey found that 50% of those who ordered onion rings on their last lunch ordered them again, while 35% switched to French fries, and 15% switched to chocolate chip cookies. Of those who ordered French fries on their first visit, 40% did so the next time, but 30% switched to onion rings, and another 30% switched to chocolate chip cookies. Of the students who ordered chocolate chip cookies on their last visit, 20% switched to onion rings, and 55% switched to French fries.

Write this information as a table and then as a matrix.

		Tomorrow		
		Onion Rings	French Fries	Choc.Chip Cookies
Today	Onion Rings	50%	35%	15%
	French Fries	30%	40%	30%
	Choc Chip Cookies	20%	55%	25%

$$\begin{bmatrix} 50\% & 35\% & 15\% \\ 30\% & 40\% & 30\% \\ 20\% & 55\% & 25\% \end{bmatrix}$$

2. On Monday, 20 students buy onion rings, 70 buy french fries, and 100 buy chocolate chip cookies. If these same students come in on Tuesday and each buy one of these items, how many orders of French fries should the manager expect to sell?

   $20(.35) + 70(.4) + 100(.55) = 90$ orders of French fries

   or using matrices:

$$\begin{bmatrix}20 & 70 & 100\end{bmatrix}\cdot\begin{bmatrix}50\% & 35\% & 15\% \\ 30\% & 40\% & 30\% \\ 20\% & 55\% & 25\%\end{bmatrix}=\begin{bmatrix}51 & 90 & 49\end{bmatrix}$$

3. If the same students come back on Tuesday, how many orders of onion rings, French fries, and cookies should the manager expect to sell?

$$\begin{bmatrix}51 & 90 & 49\end{bmatrix}\cdot\begin{bmatrix}50\% & 35\% & 15\% \\ 30\% & 40\% & 30\% \\ 20\% & 55\% & 25\%\end{bmatrix}=\begin{bmatrix}62.3 & 80.8 & 46.9\end{bmatrix}$$

which produces the same result as calculating

$$\begin{bmatrix}20 & 70 & 100\end{bmatrix}\cdot\begin{bmatrix}50\% & 35\% & 15\% \\ 30\% & 40\% & 30\% \\ 20\% & 55\% & 25\%\end{bmatrix}\cdot\begin{bmatrix}50\% & 35\% & 15\% \\ 30\% & 40\% & 30\% \\ 20\% & 55\% & 25\%\end{bmatrix}=\begin{bmatrix}62.3 & 80.8 & 46.9\end{bmatrix}$$

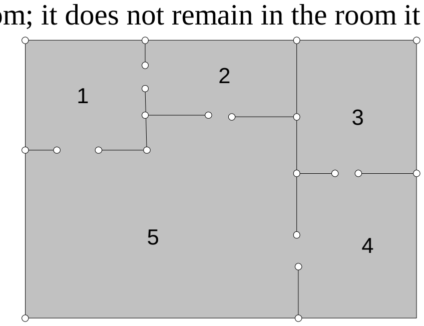
4. If these same students came back over 20 consecutive days, how many orders of onion rings, French fries, and cookies should the manager expect to sell?

$$\begin{bmatrix}20 & 70 & 100\end{bmatrix}\cdot\begin{bmatrix}50\% & 35\% & 15\% \\ 30\% & 40\% & 30\% \\ 20\% & 55\% & 25\%\end{bmatrix}^{20}=\begin{bmatrix}65.64 & 79.45 & 44.91\end{bmatrix}$$

Follow-Up Activity: The mouse

There is an animation prepared by Distance Learning staff to describe the possible movements of the mouse.

A mouse is placed in the maze shown below. During a fixed time interval, the mouse randomly chooses one of the doors available to it (depending upon which room it is in) and moves through that door to the next room; it does not remain in the room it occupies.



The following matrix describes the probability of the mouse moving from one room to another.

Move to:	room 1	room 2	room 3	room 4	room 5
room 1	0	0.5	0	0	0.5
room 2	0.5	0	0	0	0.5
room 3	0	0	0	1	0
room 4	0	0	0.5	0	0.5
room 5	0.33	0.33	0	0.33	0

If the mouse begins in room 4 its position is represented by the matrix $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.

a. What rooms might the mouse be in after one move? What is the probability for each?
b. What rooms might the mouse be in after two moves? Give the probabilities for each.
c. After 100 moves, what room is the mouse most likely to be in?

1. Using the same technique as was used in the problem above, the matrix that describes the motion of the mouse in the maze is

$$\begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.33 & 0.33 & 0 & 0.33 & 0 \end{bmatrix}$$

We know that a mouse who begins in room 4 is represented by the matrix $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.

2. The product of the two matrices will give us a matrix that describes the probability of the location of the mouse once he makes one move:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.33 & 0.33 & 0 & 0.33 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.5 & 0 & 0.5 \end{bmatrix}$$

which tells the mouse has a 50-50 chance of being in room 3 or room 5.

3. After two moves, we get

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.33 & 0.33 & 0 & 0.33 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0.166 & 0.166 & 0 & 0.667 & 0 \end{bmatrix}$$

which means the mouse is most likely in room 4.

4. For twenty moves:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.33 & 0.33 & 0 & 0.33 & 0 \end{bmatrix}^{20} = \begin{bmatrix} 0.204 & 0.204 & 0.088 & 0.221 & 0.283 \end{bmatrix}$$

which means the mouse is most likely in room 5 and least likely to be in room 3.

Student Handout
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1. The tables below represent statistics on several NFL quarterbacks for 1998 and 1999 seasons. These statistics come from http://sportsillustrated.cnn.com/football/nfl/rosters/Quarterbacks.html.

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a. On Monday, 20 students buy onion rings, 70 buy french fries, and 100 buy chocolate chip cookies. If these same students come in on Tuesday and each buy one of these items, how many orders of French fries should the manager expect to sell?

b. If the same students come back on Tuesday, how many orders of onion rings, French fries, and cookies should the manager expect to sell? Wednesday? Thursday?

c. If these same students came back over 20 consecutive days, how many orders of onion rings, French fries, and cookies should the manager expect to sell?

Follow Up Activity
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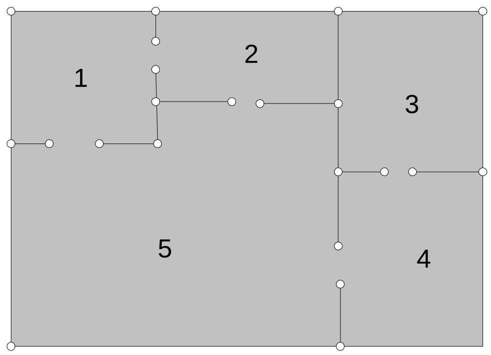

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room 5	0.33	0.33	0	0.33	0

If the mouse begins in room 4 its position is represented by the matrix $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.

d. What rooms might the mouse be in after one move? What is the probability for each?
e. What rooms might the mouse be in after two moves? Give the probabilities for each.
f. After 100 moves, what room is the mouse most likely to be in?