Relationships in Rectangles
Algebra 2

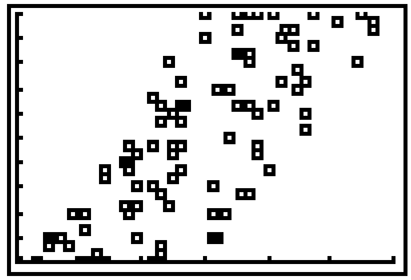
Goals:
1. Describe graphically, algebraically, and verbally real-world phenomena as functions; identify the independent and dependent variables. (3.01)
2. Write and interpret an equation of a curve (linear, exponential, quadratic) which models a set of data. (4.01)
3. Relate a restricted domain of the function with the resulting range of the function.
4. Write equations of lines and curves that form boundaries of a geometric region.

Materials Needed:
1. Copy of student handout (There is no follow up problem)
2. Ruler
3. Sheet of graph paper
4. Graphing calculator

These activities were developed based on the article "Connecting Data and Geometry" by Tim Erickson that was published in the November 2001 Mathematics Teacher.

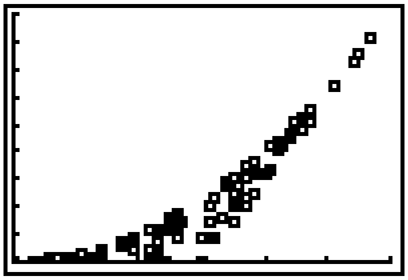
Activity One: Warm up to thinking about relationship between perimeter and area of rectangle.
1. Students will work in pairs. Each pair of students will be given an integer that represents the perimeter of a rectangle. Using the graph paper, they will sketch 5 different rectangles with this perimeter and find the area of each.
2. Based on this activity, student will make suppositions about the relationship between perimeter and area. "If *given the perimeter* of a rectangle, what can the rectangle look like? What is the area?"

Activity Two: More rectangles
Using the graphing calculator, students will create lists of random number to represent the lengths and widths of possible rectangles. Based on these measures, we will find the perimeter and area of each of the defined rectangles. By looking at graphs, relationships will be evident and we will struggle with what information is in the relationship.
1. In list L1 create 100 random integers between 0 and 30 using the command: $L1 = randInt(0,30,100)$. *RandInt* can be found under the MATH menu under PRB. This will represent the lengths of our rectangles.
2. In list L2 create 100 random integers between 0 and 30 using the command: $L2 = randInt(0,30,100)$. This will represent the widths of our rectangles.
3. Create a scatter plot of the lengths and widths. Talk about window (**this is important later**) and the visual cues that these numbers are random.
4. In L3 create a list of the perimeters using $L3 = 2L1 + 2L2$ and in L4 create a list of the areas using $L4 = L1 \bullet L2$.
5. Now we are on the hunt for relationships that describe the situation "given the perimeter of the rectangle, what is known about the length?" Create a scatter plot of length versus perimeter or the ordered pairs (perimeter, length). Consider the

window. Since lengths and widths have integer values from 0 to 30, then an appropriate window is $0 \le perimeter \le 120$ and $0 \le length \le 30$. The graph will look something like the one shown.  Remember everyone has different numbers. There is a relationship evident in this graph. What is it?

6. Ask students to look at the window of the graph to consider values of perimeter and values of lengths. Perimeter is our *x*-value since it is the given information. Can they identify any of the boundaries on the region?
    a. Using the window values (domain and range) they will connect the length as values between 0 and 30—the same is true for the widths—and the perimeters can range from 0 to 120. If the length is near zero, the perimeter values can range from 0 to 60 because $P = 2L + 2W$. If the length is near 30, the perimeter can range from 60 to 120.
    b. Points on the boundaries of the scatterplot or the vertices of the parallelogram-looking shape are (0,0), (60,0), (60,30) and (120,30). Students might use trace to determine these values or they may analyze the situations as described above in (a.).
    c. Using $P = 2L + 2W$, if the width is very small the equation is essentially $P = 2L$ or $L = \frac{1}{2}P$. Graphing $y = \frac{1}{2}x$ climbs along the left side of the parallelogram.
    d. Using $P = 2L + 2W$, if the width is near 30 the equation is essentially $P = 2L + 60$ or $L = \frac{1}{2}P - 30$. Graphing $y = \frac{1}{2}x - 30$ climbs along the right side of the parallelogram.
    e. $L = 0$ and $L = 30$ complete the boundary lines since these equations represent the restrictions on the values of the length.
    f. These equations for the boundaries tell us:
        - Given the perimeter of a rectangle, we know that the length is a value between $\frac{1}{2}$ the perimeter and $\frac{1}{2}$ the perimeter minus the width.
7. Next, we will investigate what we know about area if we are given the perimeter of the rectangle. Be prepared to be surprised! Create a scatter plot of area versus perimeter or ordered pairs of (perimeter, area). Again, talk about the window. This conversation will be useful as students try to determine the relationship.

Thinking about the values for the perimeters and the areas (given lengths and widths between 0 and 30), we find that perimeters range from 0 to 120 and areas from 0 to 900. Can we find a boundary for points that seems to follow a curve that moves along the top of the graph?



a. The point (0,0) bounds rectangles with small length and width since their perimeter is near 0 and area is between 0.
b. The point (120, 900) bounds rectangles with large length and width (near 30 for values of both).
c. Several possible strategies that students might use to determine a relationship are outlined below. However, there may be others. Good open-ended investigation.
   i. Trace along the curve and make note of several ordered pairs near the curving boundary. Go back into the lists and find these ordered pairs for (perimeter, area) in L3 and L4. Look in L1 and L2 to find the corresponding length and width of these rectangles. Write these values down. Do that for several other points on the curving boundary. Does any relationship or special situation appear? Students will see that the ordered pairs have similar values for both length and width that implies the figure is close to a square. Remember they want an equation of $area = f(perimeter)$ to test over the scatter plot. They might write: for a square with side of *s*, then $Perimeter_{square} = 4s$ and $Area_{square} = s^2$. Therefore, $area = \left(\frac{1}{4} perimeter\right)^2$. Look at the graph of $y = \left(\frac{1}{4}x\right)^2$ superimposed over the graph.

   ii. The shape of the curve appears to be a parabola with a vertex at (0,0). Therefore, the equation form is $y = ax^2$. Test out several values for a. The best curve seems to be $y = \frac{1}{16}x^2$ or $y = \left(\frac{x}{4}\right)^2$. Once this formula is determined, there must be a relationship between area and perimeter that fits this formula. Since $x = perimeter$, the formula implies the area is found by squaring the perimeter and dividing by 16. The second version of the formula says the perimeter is divided by 4 and then squared to get the area. Since $P = 2L + 2W$, if length and width were nearly equal, then $P = 4L$ or $L = \frac{P}{4}$ which leads to area as L^2 or $\left(\frac{P}{4}\right)^2$. The figure must be a square.

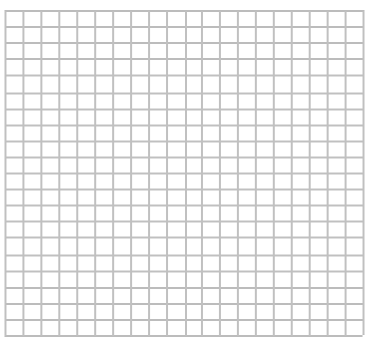
iii. A last choice for a method of finding a relationship is to use parametric equations. Using the idea that these boundary points seem to imply the rectangle is a square both perimeter and area can be written in terms of the length L. Using parametric equations we can write $x = 4t$ and $y = t^2$ to indicate that perimeter is four times a side (t) and area is a side squared. In parametric mode the ordered pairs (x,y) are plotted, which produces the desired curve.

Student Handout
Relationships in Rectangles
Algebra 2

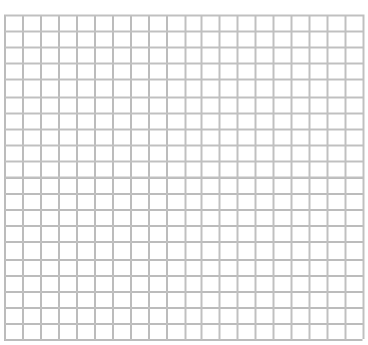
These activities were developed based on the article "Connecting Data and Geometry" by Tim Erickson that was published in the November 2001 Mathematics Teacher.

What is the relationship between the measure of the perimeter of a rectangle and the measure of its area?

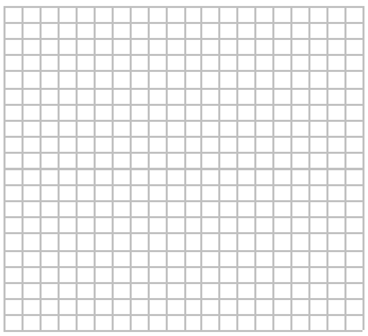
1. You and a partner will be given the perimeter of a rectangle. Using the graph paper sketch at least four different rectangles with this perimeter. Find the area for each. Given the perimeter, what do you know about the length? Given the perimeter, what can you know about area?



2. Using the random number generator of your calculator you will be given instructions to create a list of 100 random lengths and 100 random widths for rectangles. Use lengths and widths that are integers from 0 and 30. Sketch the scatter plot of the width versus the length or (length, width). Identify scale. Does the graph appear to be made up of random integers? Why?



3. Given the perimeter of a rectangle, can you determine anything about the length? In list L3 create the perimeters of the rectangles whose lengths and widths are listed in lists L1 and L2.
   a. Sketch the scatter plot of length versus perimeter or (perimeter, length). Label scale.
   b. This scatter plot seems to have boundaries. What are the boundaries and what meaning do these boundaries have for length and perimeter?



4. Given the perimeter of a rectangle, what can you know about the area? In list L4 create the areas of the rectangles whose lengths and widths are listed in lists L1 and L2.
   a. Sketch the scatter plot of area versus perimeter or (perimeter, area). Label scale.
   b. This graph seems to have a curved boundary. What is this boundary and what meaning does it have to the question of area and perimeter?