Algebra 2
The Box Problem

Goals:
- Translate among graphic, algebraic, and verbal representations of relations (3.02)
- Graph relations and functions and find the zeros of function. (3.03)
- Use polynomial equations to solve problems. Solve by graphing. (3.07)
- Find zeros, intercepts, and approximate turning points of polynomial functions; describe them in the context of the problem. (3.08)
- Write and interpret an equation of a curve which models a set of data.(4.01)

Materials and Equipment Needed:
- Graphing calculator
- Ruler (one for every two students)
- Scissors (one pair for each group)
- Scotch tape
- Sheet of cardstock (8.5 by 11) for each group
- There is **no** student handout for the main problem
- Copy of follow up problem handout for each student (optional).

Activity One: Building an open-top box from a sheet of cardstock

1. Groups build a specified box. There is an animation video in the DL studio that demonstrates how to build the boxes.
   - Separate students into groups or pairs.
   - Each group/pair receives a sheet of cardstock, one or two rulers, scissors, and tape.
   - Students are given the dimensions of the square that will be cut from each corner of the cardstock. Squares with sides ranging from 0.5 inch to 4 inches will give good results.
   - Fold the cardstock into a open-top box.
   - Find the volume of the box. Most likely students will find this volume by measuring with the rulers. This is what we want. If they begin to use a formula on their own, don't discourage them, but don't suggest it either.
   - Table of volumes (the students may not get these exact values)

x-value	0.5	1	1.5	2	2.5	3	3.5	4
volume	37.5	58.5	66	63	52.5	37.5	21	6

2. Collect data from the groups
   - Ask each group to report on the length of the size of the square cut from the corner of the paper and the volume (*volume=length*width*height*) of the box created. A list will be made that all students can put into their calculator lists.
   - Make a scatter plot of the data with ordered pairs of the form (length of side of square, volume).
   - Look at the point that produces the box of most volume (1.5, 66) and of least volume (4,6).
   - Pose the question of whether these points represent the boxes that produce most volume or least volume of *any* box built from this sheet of cardstock. Students might have noticed that the volumes increase to a point and then begin to decrease. Using this information they could narrow the lengths of the square sides that might produce a larger box, however, to continue to make paper boxes is a tedious task.

3. Develop a function whose ordered pairs represent the ordered pairs (length of side of square, volume) of all boxes that can be built from this paper.
   - Write an expression that represents the volume of the box when the length of the side of the square removed is x. *Volume=length*width*height*, so $Y1 = (11 - 2*x)(8.5 - 2*x)*x$.
   - Graph this equation over the scatter plot of the data collected by the class. There might be some discrepancies since the volumes were most likely found by measuring and some error is likely to occur.
   - **Determine the dimensions of the box with greatest volume**. You can find this value on the TI-83 graphing calculator with the MAXIMUM option. With the graph and scatter plot on the calculator screen, press 2nd TRACE, then select option 4. Move the cursor somewhere to the left of the maximum value and press ENTER. Move the cursor somewhere to the right of the maximum value and press ENTER. Move the cursor somewhere close to the maximum value and press ENTER. The calculator will give the values $x \doteq 1.59$ and $y \doteq 66.15$. Remind the students that this is telling them that if they cut out a square of side length approximately 1.6 inches they will have a box that holds approximately 66.15 inches^3. This is the largest box.

   - **Determine the dimensions of the box with a specified volume**. Suppose we want to know how large a square to cut out to get a box that holds 40 inches^3? We can look on our calculators at the intersection of our *volume* function (Y1) and the function Y2=40. With both functions (Y1 and Y2) showing on the calculator screen, press 2nd TRACE, then select option 5. This works much like the MAXIMUM option from above. We will have a box that holds 40 inches^3 when we have squares with sides of approximate length 0.54 inches and 2.9 inches. It is helpful to have made these two boxes ahead of time so the students can see how different they are, yet they will hold the same amount.

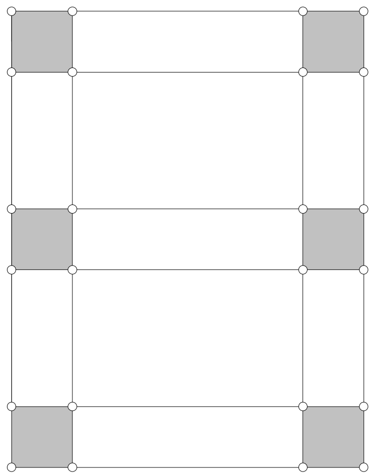
   - **Determine the dimensions of the box with least volume in our class**. We can also discuss the meaning of negative values of the function. This brings up the issue of <u>domain.</u> The function $Y1 = (11 - 2*x)(8.5 - 2*x)*x$ has a domain of all real numbers when it is not in a context. However, when we are talking about building boxes, the domain is restricted to $0 \le x \le 4.25$. The endpoint is zero because we can't cut out a square of side length less than zero. The 4.25 is the upper limit because we can't cut out a square of side length greater than 4.25 since the short side of our paper is 8.5 inches long – you can't go further than half way and still cut out squares. If we think about building a box with a volume of 0, this will lead us to the zeros of the function that connect to the factors in our volume function.

Follow-Up Activity: Students try a similar problem.
On the handout, students are given another method of building a box. This design produces a box with a lid.

1. Students work in small groups or pairs to answer the questions.
2. The teacher may provide additional cardstock to build this new box. This is optional. The task of building this box is as much a challenge as investigating the volume!
3. Answers:
   - The function would be *volume=length*width*height*, so $Y1 = \frac{(11-3x)}{2} * (8.5 - 2x) * x$
   - The x-values that produce zero volume would correspond to the factors in the volume equation, so $x = 0$ and $x = 11/3$ would produce boxes with zero volume. Although $x = 4.5$ is also a zero of the function, it is beyond the domain of interest in this setting.
   - The x-value that would produce a box with the greatest volume is $x = 1.3$ inches that produces a box with approximate volume 27.2 inches^3.

Follow-Up Activity
Algebra 2
Another Box Problem

1. In the figure shown below, squares are cut from a sheet of cardstock paper. The length of the side of each square is x. Once the squares are cut out, the paper is folded into a box with a lid.



a. Write a function that describes the volume of the box in terms of the length of the side of each square represented by x.

b. Determine the value of x that will produce a box of the least volume. What is the volume for this value of x? Document your work.

c. Determine the value of x that will produce a box of greatest volume. What is the volume for this value of x? Document your work.