Voltage Data Collection for Exponentials
Algebra 2

Goals:

1. Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables. (3.01)
2. Translate among graphic, algebraic, and verbal representations of relations. (3.02)
3. Write and graph exponential functions of the form $f(x) = a \cdot b^x$. (3.15)
4. Use logarithmic and exponential functions to solve problems. Solve by graphing, substitution, applying the inverse relationship, using properties of equality. (3.17)
5. Write and interpret an equation of a curve (exponential) which models a set of data. (4.01)
6. Find the equation of best-fit (exponential) for a set of data. Interpret the constants, coefficients, and bases in the context of the data. Check the equation for goodness-of-fit and use the equation for predictions.
7. Use exponential equations of the form $f(x) = (1+r)^x$ where r is given as a rate of growth or decay to solve problems. (4.03)

Materials Needed for Each Student:

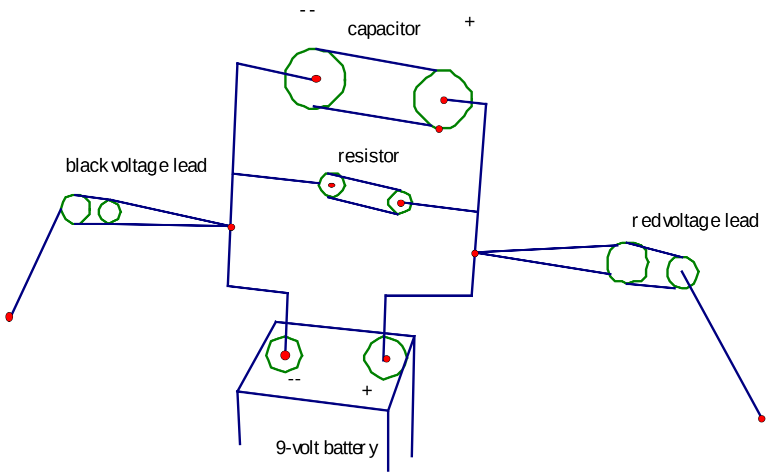
1. Student Handout
2. Graphing calculator

If performing the experiment each group will need:

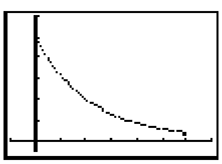
3. CBL and link to calculator
4. Voltage probe for CBL
5. Nine volt battery
6. 220-microfarad capacitor and 100-kohm resistor (found at Radio Shack as RS#272-1017 and RS#271-1347). A nice picture of the set up is shown on http://www.math.montana.edu/frankw/ccp/CBL/voltage/TI-83/text.htm. The positive end of the capacitor has a small ridge around the cylinder, use red for positive and connect all positives together.
7. Volt Program for the TI-83. (If using the CBL2, the program is there in the Datamate program.)

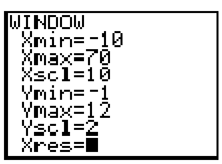
Activity One: Taking Data

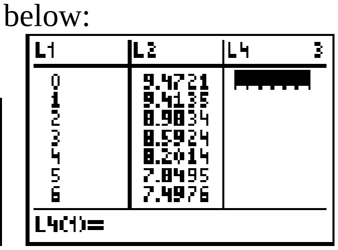
Connect red voltage lead on the positive end of the capacitor below where the resistor attaches and the black voltage lead on the negative end in the same way.



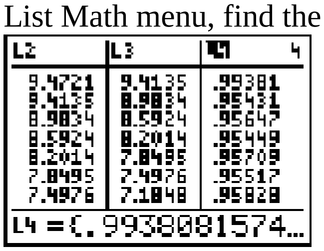
Hold the capacitor/resistor/leads piece so the wires touch the appropriate (positive/negative) battery posts. With the voltage probe in the Channel 1 plug of the CBL and the CBL connected to the calculator, run the voltage collection program. It is best to set the program to collect voltage measures every second for 60 seconds. When the program begins, immediately remove the capacitor wires from the battery so the voltage will discharge. Time will be recorded in L1 and volts in L2. The scatter plot should show the voltage decreasing over time.
A sample data collection is shown below:

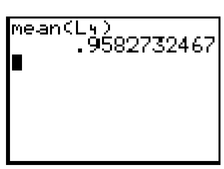






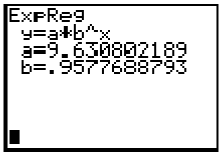
To attempt to understand the relationship in the data, find the ratio of the voltages where V represents the voltage and t represents time. Divide $\frac{V(t+1)}{V(t)}$ or take the ratio of a voltage reading and the voltage reading of the previous second. We are trying to see if the voltage falls by a consistent percentage every second—which would describe exponential decay.
To accomplish this, copy the data from L2 in L3. Delete the first entry in L3 and the last entry in L2. In list L4, find $\frac{L3}{L2}$. Each entry in L4 is the percentage drop from $V(t)$ to get $V(t+1)$. Each entry in L4 will not be exactly the same, but will be very close. Using List Math menu, find the Mean(L4).





This mean represents on average, the percentage drop in voltage from one second to the next is 95%.

Now find the exponential regression curve through the data by ExpReg L1, L2, Y1. (Delete the last entry in L1 so the L1 and L2 will be the same length. Remember you deleted a value from L2.)



The base of the exponential function and the mean (L4) should be close in value. Since the battery was a 9-volt battery, the voltage reading at initial time should be 9. Notice that our data has a bit higher reading and the value of *a* in the function $y = a * b^x$ is larger than 9 as well. We should expect both to be near 9 in value.

Activity 2: Hurricane Fran

Hurricane Fran hit North Carolina on the evening of September 5, 1996. Over one million homes and businesses were left without power. Repair crews began immediately restoring electrical service. What is the average rate of customers without power who remain without power the next day? Compare this value to the base of the exponential function that fits this data. (This data is taken from the Algebra 2 Indicators prepared by the NC Department of Public Instruction.)

Date	Customers without power
Sept 6	1,159,000
Sept. 7	804,000
Sept. 8	515,000
Sept. 9	340,500
Sept. 10	195,200
Sept. 11	136,300
Sept. 12	77,000
Sept. 13	37,600

a) Find the average (mean) of the ratios of the customers without power on one day with the customers of the previous day for everyday.
b) Find the equation of the best-fit exponential function that fits this data. Be sure to convert the list of days of September to days since September 6.
c) Compare the value of the mean of the ratios with the base of the exponential function.

1. Using the list feature of the graphing calculator, put date in September in L1 and the number of customers without power in L2. Convert L1 to days since September 6 by using the formula L1=L1-6 at the heading of L1.
2. Copy the list of values of L2 into L3 by using the formula L3=L2 at the heading of L3. Delete the top element from L3 and the last element from L2. Use L4 to find the

ratio using the formula L4=L3/L2. Have students look over the values in L4; there is quite a variety.

3. Find the mean (L4) on the home screen of the calculator. The value is 0.6172
4. Next find the ExpReg for L1 and L2, but remember to re-enter in the deleted value into L2. The result is $y = 1306850(0.61914)^x$.
5. The value of the mean of the ratios is 0.6172 while the base of the exponential function is 0.6191—quite close.

Follow-Up Activity: **North Carolina Population**

The North Carolina population is counted with every census. In the table below, the populations are given for every other census. What is the average rate of growth for the population of North Carolina? What is the time period for this rate?

year	population (to nearest 1000)
1800	478,000
1820	639,000
1840	753,000
1860	993,000
1880	1,400,000
1900	1,894,000
1920	2,559,000
1940	3,572,000
1960	4,556,000
1980	5,880,000
2000	8,049,000

This problem is taken from the Algebra II Indicators developed by the NC Department of Public Instruction.

1. Put years in L1 and population in L2. Change L1 to years since 1800 by using the formula L1=L1-1800.
2. Copy list L2 into list L3. Delete the top entry of L3 and the bottom entry of L2 and find the ratio of each entry in L4 using the formula L4=L3/L2.
3. Find the mean of elements of L4. The value of the mean is 1.328. Note the values in L4 are quite large. The values are above 1 since the population is growing and the time interval is 20 years so values show rate of change for 20-year periods.
4. Find the ExpReg of values from L1 and L2. Be sure to re-enter the deleted value form L2. The result is $y = 453{,}635(1.0144)^x$.
5. The comparison of the mean of the L4 values and the base of the exponential function are very different. However, when the time intervals for the two are adjusted to be the same, the comparison shows a stronger correlation. The exponential function base can be changed to a twenty-year rate by finding the value of $(1.0144)^{20}$ which is 1.331. This is very near the mean (L4). Also the twenty-year rate of mean (L4) can be converted to a yearly rate by find the value of $\sqrt[20]{1.328}$ which is 1.0143.

Student Handout
Algebra 2
Voltage Data Collection

1. Voltage Data Collection

2. Hurricane Fran hit North Carolina on the evening of September 5, 1996. Over one million homes and businesses were left without power. Repair crews began immediately restoring electrical service. What is the average rate of customers without power who remain without power the next day? Compare this value to the base of the exponential function that fits this data.

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