Money and the Exponential Function
Algebra 2

Goals:

1. Write and graph exponential functions of the form $f(x) = a \cdot b^x$ (3.15)
2. Use exponential equations to solve problems. Solve by graphing, substitution. (3.17)
3. Use exponential equations of the form $f(x) = (1+r)^x$ where *r* is given as a rate of growth or decay to solve problems. (4.03)
4. Find the Annual Percentage Rate (APR) for a given compounded rate.

Materials and Equipment needed by each student:

1. Copy of the student handout
2. Calculator
3. Paper and pencil for note taking.

Activity One: Compound Interest

A friend borrows $300 from you for maximum of thirty days. You charge your friend either1% per day or 30% (once) for entire 30 days for the use of your money. Compare the amounts your friend owes you.

1. Define compound interest as interest gained on the balance (amount borrowed and past interest).
2. For the 30%: Calculate $\$300(1+0.30) = \$390.00$
3. On compounded interest: Evaluate $\$300(1+0.01)^{30} = \$404.35$
4. Clearly the borrower would like the 30% and the lender would like 1% per day.
5. What is the full 30-day rate that is the same as 1% per day for 30 days? Evaluate: $(1+0.01)^{30} = 1.3478$ which shows a rate of 34.78% for the full 30 days.

Activity Two: Savings Account

Lindsey's grandmother gave her $1000 on the day she was born. Lindsey's parents put the money into a savings account that earns 6.4% interest each year.
a. How much money will Lindsey have in the account on her 18th birthday if the interest is compounded annually, quarterly, monthly, daily, hourly, each minute, each second, continuously?
b. In banking, the term Annual Percentage Rate (APR) is used to give the value of the annual simple interest rate that is equal to the compounded rate of an account. What is the APR for 6.4% each of the compounding schemes above?

1. Discuss the meaning of compounding using the formula $A = A_0\left(1+\frac{r}{n}\right)^{nt}$, where

   $A_0 =$ the initial investment, $r =$ the annual rate, $n =$ number of times compounded in the year, and $t =$ number of years. Discuss the values of each of these variables in this problem.

2. Use the 2nd Enter aspect of the calculator to re-enter the equation just used and then by editing find the value for each of the 18 year compounding methods.
3. To find the APR, find the value of $\left(1+\frac{r}{n}\right)^n$. This value will determine the amount that the principal is multiplied by each year or by subtracting 1, gives the annual rate the principal is multiplied by each year.

Compounded	Formula: $A = A_0\left(1+\frac{r}{n}\right)^{nt}$	Value	APR
annually	$\$1000\left(1+\frac{.064}{1}\right)^{1\cdot 18}$	\$3054.56	0.06400
quarterly	$\$1000\left(1+\frac{.064}{4}\right)^{4\cdot 18}$	\$3135.79	0.06555
monthly	$\$1000\left(1+\frac{.064}{12}\right)^{12\cdot 18}$	\$3154.84	0.065911
daily	$\$1000\left(1+\frac{.064}{365}\right)^{365\cdot 18}$	\$3164.19	0.066086
hourly	$\$1000\left(1+\frac{.064}{365\cdot 24}\right)^{365\cdot 24\cdot 18}$	\$3164.50	0.066092
every minute	$\$1000\left(1+\frac{.064}{365\cdot 24\cdot 60}\right)^{365\cdot 24\cdot 60\cdot 18}$	\$3164.51	0.066092
every second	$\$1000\left(1+\frac{.064}{365\cdot 24\cdot 60\cdot 60}\right)^{365\cdot 24\cdot 60\cdot 60\cdot 18}$	\$3164.46	0.066092
continuously	$\$1000\cdot e^{0.064\cdot 18}$	\$3164.51	0.066092

Note the technology begins to fail with the evaluation of compounding every second. This result is probably not reliable.

Activity Two point Seven: What is e?

One dollar is invested at 100% interest per year for a year. How much money is in the account at the end of the year? What value does the amount in the account approach as the number of compounding periods increases?

1. One dollar invested at 100% interest per year for a year would yield $\$1(1+1)^1 = \$2$.
2. The table below will illustrate the value that the account approaches:

Compounded	formula	amount
Yearly	$\$1(1+1)^1$	\$2
Monthly	$\$1\left(1+\frac{1}{12}\right)^{12}$	\$2.61303529

Daily	$\$1\left(1+\frac{1}{365}\right)^{365}$	\$2.71456748
Hourly	$\$1\left(1+\frac{1}{365\cdot 24}\right)^{365\cdot 24}$	\$2.71812669
every minute	$\$1\left(1+\frac{1}{365\cdot 24\cdot 60}\right)^{365\cdot 24\cdot 60}$	\$2.71827922

The value of $e = 2.718281828$ which helps us see the definition of $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$.

Follow Up: Several Applications

The following questions from the Algebra 2 Indicators prepared by the NC Department of Public Instruction.

1. The number of airline passengers increased from 465.6 million in 1990 to 614.3 million in 1998.
   - What was the average annual growth rate (percent) for the 1990 to 1998 period?
   - If that rate remains constant after 1998, how many airline passengers can be expected in 2005?
   - Give the algebraic model for this growth.
2. At the end of four years (**t**), a savings account paying 5.35% annually (**r**) compounded continuously had a balance (**B**) of \$3096.56. What was the initial deposit (**P**)? (Use $\mathbf{B} = \mathbf{Pe}^{rt}$) If the initial deposit had been in an account compounded annually, how much less interest would have been earned?
3. Solve $350 = 200e^{2r}$ for r. Justify each step.
4. Solve $960 = A\cdot 1.075^{8}$ for A. Justify each step.
5. Solve $663 = 49\cdot 2.165^{x}$ for x. Justify each step.

Each of these problems is simplified by using the graphing calculator.

1. In this question, the two points can become ordered pairs with the *x*-values in L1 and the *y*-values in L2. Use 0 for 1990 and 8 for 1998. Using ExpReg from the Stat Calc menu, the calculator produces the function $y = 465.6(1.03525)^{x}$. Since *x*-values are years, we know that each year the number of riders is multiplied by 1.03525. This creates an annual growth rate of 3.525% each year. To find the number of passengers in 2005, find the value of Y1(15) since 2005 is 15 years from 1990. This result is 782.9, which is the number of passengers expected in 2005. The algebraic model is $y = 465.6(1.03525)^{x}$.

2. When we substitute the values given into the equation, the result is $3096.56 = Be^{0.0535\bullet 4}$. Solve this by finding the value of $e^{0.0535\bullet 4}$.

$$3096.56 = Be^{0.0535\bullet 4}$$
$$3096.56 = B\cdot 1.238622655$$
$$\frac{3096.56}{1.238622655} = B$$
$$2500.00 = B$$

With annual compounding, the equation would be $P = 2500\left(1+\frac{0.0535}{1}\right)^4 = 3079.49$

This will result is $17.07 less than with continuous compounding.

3. Graph $y = 350$ and $y = 200e^{2r}$ on the same window. Use the intersect option on the calculator. A window that shows the intersection is $0 < x < 5, 0 < y < 400$. The point of intersection occurs at $(0.2798, 350)$. Therefore, $r = 0.2798$

4. By finding the value of 1.075^8 and substituting this value into the equation,

$$960 = A\cdot 1.075^8$$
$$960 = A\cdot 1.783477826$$
$$\frac{960}{1.783477826} = A$$
$$538.274 = A$$

5. In this question, graph two functions $y = 663$ and $y = 49\cdot 2.165^x$. A window that shows the intersection is $0 < x < 5, 0 < y < 800$. The point of intersection occurs at $(3.37246, 663)$. Therefore, the solution is $x = 3.37246$.

Student Handout
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hourly			
every minute			
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