The Drug Problem, Introduction to Exponential Functions
Algebra 2

Goals:
1. Describe graphically, algebraically and verbally real-world phenomena as functions; identify independent and dependent variables. (3.01)
2. Translate among graphic, algebraic, and verbal representations of relations. (3.02)
3. Write and graph exponential functions of the form $f(x) = a \cdot b^x$. (3.15)
4. Use exponential equations to solve problems. Solve by: a) Graphing. (3.17)
5. Write and interpret an equation of a curve (exponential) which models a set of data. (4.01)
6. Find the equation of the curve of best-fit (exponential) for a set of data. Interpret the constants, coefficients, and bases in the context of the data. Check the equation for goodness-of-fit and use the equation for predictions. (4.02)
7. Use exponential equations for the form $f(x) = a \cdot (1+r)^x$, where r is given as a rate of growth or decay to solve problems. (4.03)

Materials and Equipment Needed:
1. Copy of Student Handout for each student
2. Calculator

Activity 1: How much medicine is in the body?
Activities were taken from <u>Drugs and Pollution in the Algebra Class</u>, James T. Sandefur, *The Mathematics Teacher*, Volume 85, No. 2, February 1992.

Assume that a dose of some medicine is taken internally. We are going to study how that medicine is eliminated from the body. We add 1 quart (4 cups) of water to the container to model the blood in the body. We assume that the proper dose of this medicine, say, cough syrup, is 16 mL. To model this amount, we add 16 mL of food coloring to the container.

A simplistic description of how the kidneys remove medicine from the bloodstream is that during any fixed time period, say, a four-hour period, the kidneys take in a fixed percent of the blood and remove the medicine from the blood. We will assume the kidneys purify one-fourth of the blood during any four-hour period. Assume no additional doses are taken.

a) We begin with 1 quart of water and 1 16 mL dose of cough syrup. Each 4-hour period one-fourth of the cough syrup is removed and replaced by clean water. After one day, or six 4-hour periods, how much medicine is left in the body?
b) Using your graphing calculator, make a scatter plot of amount of medicine left in the body versus four-hour time periods.
c) Find the model of best fit for the scatter plot. Check the residuals to verify you have a good fit.
d) Predict how much medicine will be left in your body after 2 days. after 3 days. after 5 days. after 1 week.

e) Predict when there will be 1 mL of cough syrup left in your body.

Some Solutions.

a)

4 hr time periods	0	1	2	3	4	5	6
amount of medicine (mL) left in the body	16	12	9	6.75	5.0625	3.796875	2.84765625

Focus on how to get these values using your calculator.

$$16-0.25*16=12$$
$$12-0.25*12=9$$
$$9-0.25*9=6.75$$
$$\vdots$$
$$ANS-0.25*ANS=amount\ left$$

b) Look at the scatter plot of the 7 ordered pairs from above. They are of the form $(time, amount)$. Set the window on the calculator so the axes are visible. A setting of $-1 \le x \le 7$ and $-1 \le y \le 20$ is a good choice. Point out to the students that the initial amount is obvious from the scatter plot from the $y-$intercept. Ask the students what they expect the graph to look like as time continues to pass. They should expect it to continue to drop. Ask if they expect it to ever get to zero. Define *horizontal asymptote* to be the $y-$value that is approached but never reached. An exponential function has this characteristic. An exponential function is one of the form $y=a*b^x$. Notice that the independent variable, x, is the exponent, a is the leading coefficient, and b is the base.

At this point there probably needs to be some discussion of the differences between exponential and quadratic functions. The equations $y=x^2$ and $y=2^x$ can be compared from the algebraic (compare table of values) and graphical (compare graphs) points of view.

c) Using the ExpReg option on the graphing calculator, students can find the best-fit exponential function to be $y=16*0.75^x$. What does the 16 tell you? It is the initial amount of cough syrup in the body. What does the 0.75 tell you? Three-fourths of the medicine is staying in the body each time period (one-fourth is being eliminated each time period).

The residuals can be calculated using the expression $L3=L2-Y1(L1)$ given that $L3$ is where you are storing the residual values, $L2$ is where the $y-$values are stored, $L1$ is where the $x-$values are stored, and $Y1$ is where the linear regression model is stored. The residual values are all equal to zero. Point out to students that this is the case since the $y-$values were generated using an equation and are not coming from a real-world situation where data was gathered.

d)

$y = 16 * 0.75^{12} \approx 0.5068$

$y = 16 * 0.75^{18} \approx 0.0902$

$y = 16 * 0.75^{30} \equiv 0.0029$

$y = 16 * 0.75^{42} \approx 0.0000905$

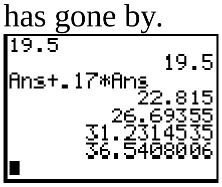
Reinforce how these values imply there is an asymptote.

e) Graph the exponential function $y = 16 * 0.75^x$ along with the function $y = 1$ and find the intersection to be approximately $(9.637, 1)$. There will be 1 mL of cough medicine left in the body after approximately $9.637 * 4 \approx 38.6$ hours. Students will need to adjust their calculator window to see the intersection point. A good choice is $-1 \le x \le 15$ and $-1 \le y \le 3$.

Activity 2:

Over the last year the stock value of an internet company has grown at a rate of 17% per month. The value of the stock at the beginning of the year was $19.50. What was the value of the stock at the end of the year? If the stock's value continues to grow at the same rate, how long does it take an investor to triple her money? Taken from the Algebra II Indicators from NC Department of Public Instruction.

1. This problem is similar to the drug problem. On the home screen of the calculator, we can begin with $19.50. Using ANS key, we can look at the value of the stock each month by using $ANS + 0.17 \cdot ANS$. Every time we hit the enter button, a month has gone by.

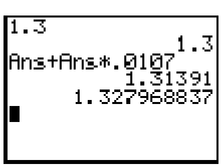


The result is that the stock is worth $128.31 at the end of the year.

2. This list also allows us to consider when the value has tripled. Three times $19.50 is $58.50. Looking at the result after one year, we know that the stock tripled during that year. By doing the same process again and watching for a value of 58.5 tells us, the stock tripled in value during the seventh month.

Follow-Up Problems

1. The population of China is growing annually at a rate of 1.07%. The last census was completed in the year 2000 and the population was 1.3 billion. When will the population reach 2 billion?



After 41 enters or 41 years, the population is above 2 billion. You could also create a function $P(x) = 1.3(1 + 0.0107)^x$ and graph this function with $y = 2$ to find the point of intersection.

2. For most people, the half-life of caffeine is 7 hours. Seven hours after eating some amount of caffeine, you will have one-half of that amount in your body. If you drink a cup of coffee at 7 a.m., the coffee introduces 5 milligrams of caffeine into your body. How much caffeine will be in your body when you go to sleep that night? Every 7 hours the amount is cut in half. The following table gives values of the caffeine content in the body:

time	7 am	2 pm	9 pm	4 am
caffeine	5 mg	2.5 mg	1.25 mg	0.625 mg

We will have between 1.25 and 0.625 mg in the bloodstream when we go to sleep.

Student Handout
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4 hr time periods							
amount of medicine (mL) in the body							

b) Using your graphing calculator, make a scatter plot of amount of medicine left in the body versus four-hour time periods.
c) Find the model of best fit for the scatter plot. Verify that you have a good model by looking at the residual values.
d) Predict how much medicine will be left in your body after 2 days. after 3 days. after 5 days. after 1 week.
e) Predict when there will be 1 mL of cough syrup left in your body.

2. Over the last year the stock value of an internet company has grown at a rate of 17% per month. The value of the stock at the beginning of the year was $19.50. What was the value of the stock at the end of the year? If the stock's value continues to grow at the same rate, how long does it take an investor to triple her money? Taken from the Algebra II Indicators from NC Department of Public Instruction.

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