

## Algebra 2

### Compositions and Inverses of Functions

#### Goals:

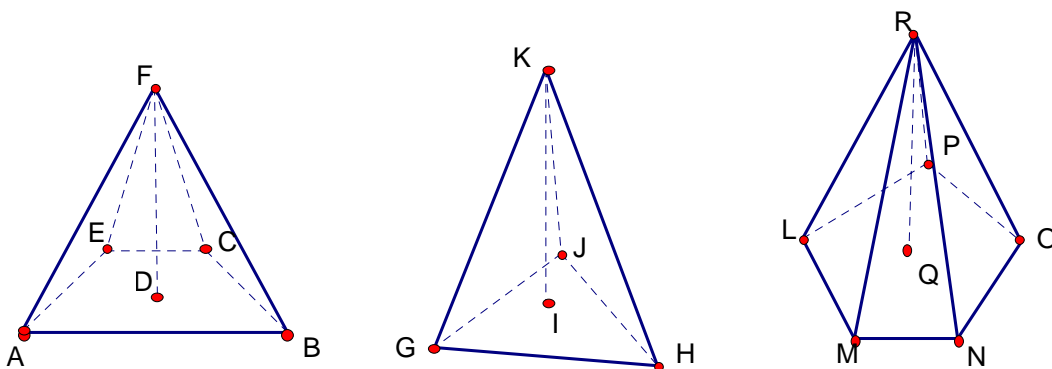
1. Operate with algebraic expressions to solve problems (1.02)
2. Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables (3.01)
3. Translate among graphic, algebraic, and verbal representations of relations (3.02)
4. Find the composition and inverse of functions (3.03)
5. Write and interpret an equation of a curve that models a set of data (4.01)
6. Find the equation of the curve of best-fit for a set of data. Interpret the constants, coefficients, and bases in the context of the data. Check the equation for goodness-of-fit and use the equation for predictions.

#### Materials needed for students:

1. Copy of student handout
2. Graphing calculator

#### Activity 1: Composition of functions in finding volumes

The volume of a pyramid is found using the formula  $\text{Volume} = \frac{1}{3}(b) \cdot h$ , where  $b =$  area of the base and  $h =$  height of the pyramid. In this example we will consider the value of  $h$  to be constant. Bases of pyramids are plane figures; for example, triangles, quadrilaterals, or pentagons. Consider the following figures and find the formula for the volume of the pyramid.



- a) Figure ABCEF is a pyramid with a square base. If the length of  $AB = x$  and  $DF = h$ , find the volume of the pyramid.
- b) Figure GHJK is a pyramid with an equilateral triangle for the base. If the length of  $GH = x$  and  $KI = h$ , find the formula for the volume of this pyramid.
- c) Figure LMNOPR is a pyramid with a regular pentagon for the base. If the length of  $MN = x$  and  $QR = h$ , find the formula for the volume of this pyramid. [Hint: the area of a regular pentagon is given by the formula  $A = \frac{5}{4} \tan 54^\circ x^2 \approx 1.72x^2$ .]

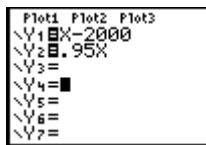
- d) For each of these volumes, consider your procedure. Use composition of functions to describe procedure.
- In this activity, students will be remembering formulas from geometry and may need some “nudges” to remember. One of the parts of a, b, and c could be assigned to different groups or rows of seats. The important issue is the procedure.
  - In each case students are taking the formula for the area of the base, which is a function of the length of the side (since all figures are regular) and then substituting that formula into the volume formula for  $b$ . To add some symbolism to the process, look at each figure.
  - First look at the equilateral triangle. We know that  $b(x) = \frac{\sqrt{3}}{4}x^2$ . Since  $V(b) = \frac{1}{3}b \cdot h$ , we can see that  $V(b(x)) = \frac{1}{3} \cdot b(x) \cdot h = \frac{1}{3} \cdot \frac{\sqrt{3}}{4}x^2 \cdot h$ . In this expression, we are treating  $h$  like a constant.
  - In the pyramid with the square base, we know that  $b(x) = x^2$  and  $V(b) = \frac{1}{3}b \cdot h$ .  
Therefore,  $V(b(x)) = \frac{1}{3} \cdot b(x) \cdot h = \frac{1}{3} \cdot x^2 \cdot h$ .
  - In the pyramid with the pentagonal base, we know that  $b(x) = \frac{5}{4} \tan 54^\circ x^2$ .  
Therefore,  $V(b(x)) = \frac{1}{3} \cdot b(x) \cdot h = \frac{1}{3} \cdot \frac{5}{4} \tan 54^\circ x^2 \cdot h$ .

### Activity 2: Car Deals.

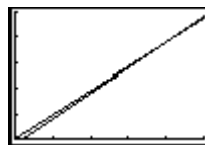
North Car Sales traditionally sells fewer cars in the months of January, February, and March. This year they will try several initiatives to sell more cars. In January, they will offer a rebate of \$2000 on every car sold. In February, the price of every car will be reduced by 5%. In March, to really get things going they will offer both the deals of January and February.

- For January and February, write a function to determine the discounted cost of a car where  $C$  = the original cost of the car before discount. Use function notation to write these functions where January is represented by  $J(C)$  and February is represented by  $F(C)$ .
  - Determine whether it is best to buy in January or February if the costs of the cars are between \$5000 and \$50,000.
  - When the two deals are combined in March, there are two possible function compositions. Find the two possible functions for the discounted price in March.
  - Which gives the best deal of the two compositions? Explain.
  - Would you wait to buy a car in March rather than January or February? Why?
- The function for January would be  $J(C) = C - 2000$  and the function for February would be  $F(C) = 0.95C$ .

2. The function that gives lowest price for the car changes over the domain. If  $C < 40000$ , the best deal is in January. If  $C > 40000$ , the best deal is in February. This can be investigated with either table or a graph on the calculator. If  $C = 40000$ , either deal is the same.



X	Y1	Y2
0	-2000	0
10000	8000	9500
20000	18000	19000
30000	28000	28500
40000	38000	38000
50000	48000	47500
60000	58000	57000



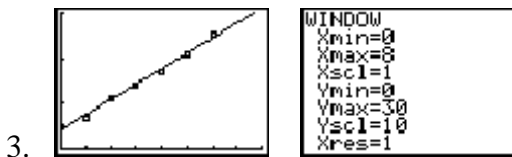
3. The two possible function for March are  $M_1(C) = J(F(C)) = (0.95x) - 2000$  or  $M_2(C) = F(J(C)) = 0.95(x - 2000)$ . Compare graphs or tables on the calculator or use the distributive property to determine the function  $M_1$  gives lowest prices. The function  $M_2$  has the multiplication of 0.95 acting on both  $x$  and on \$2000, which lowers the amount subtracted.
4. I would wait until March to purchase a car. Using the graph or table of values will show that March is much cheaper.

### Activity 3: How is an inverse function useful?

A survey is given to students in which the student states the average number of hours per week they study and the number of honors-level or AP courses they take. The average values for each course load is shown in the table.

Honors/AP Courses	0	1	2	3	4	5	6
Hours of Study each week	5	7	11	14	17	21	25

- Determine the independent and dependent variable.
  - Using data analysis methods, find a linear equation that will serve as a good model for this information. Superimpose this function over the scatter plot of the data.
  - Interpret the meaning of the slope of the line in the context of this problem.
  - Suppose a student uses his or her study habits to determine the number of honors-level or AP courses to take. What would be the independent variable? Using the concept of the inverse of a function, find the equation of the inverse of the linear function found above. Interpret the slope of this line.
  - Graph both functions on the same coordinate system. Discuss how to select an appropriate graphing window. Discuss domain and range for each function.
  - If Frank knows he has only 15 hours per week to put toward his studies, determine the number of honors-level or AP courses he should take, according to this model. Discuss your method.
- The scatter plot shows a function that appears linear. The independent variable is number of courses and the dependent variable is hours of study per week.
  - The equation that best fits the data is  $y = 3.357x + 4.214$  or  $h(x) = 3.357x + 4.214$ .



4. The slope means for every course added the hours per week of study increases by 3.3 hours.
5. When we become focused on thinking of the hours we study in order to determine the number of courses we should take, we are thinking of hours at the independent variable and courses as the dependent variable.
6. To find the inverse of the function, solve for the  $x$  in  $y = 3.357x + 4.214$ .

$$y = 3.357x + 4.214$$

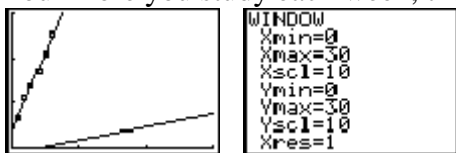
$$y - 4.214 = 3.357x$$

$$\frac{y - 4.214}{3.357} = x$$

$$0.30y - 1.25 = x$$

The inverse function is  $h^{-1}(x) = 0.30x - 1.25$ .

The slope of this line bears the units of courses/hour. The slope means that for every hour more you study each week, the number of courses increases by 0.3.



The window for the graph that shows both equations usually will have the same range of values for both  $x$  and  $y$ .

7. For Frank who can study 15 hours each week, the number of courses he can take can be determined in two ways:
  - find the values of  $h^{-1}(15) = 3.21$  or 3 courses
  - in the equation  $y = 3.357x + 4.214$ , let  $y = 15$  and solve for  $x$ . This also produces the value 3.21 or 3 courses.

### Follow-Up Activity

1. You can see a greater distance from a higher building. A function that describes the distance one can see from a tall building on a clear day is  $D(h) = 1.2\sqrt{h}$  where  $h$  = height in feet and  $D$  = distance you can see in miles. You can find information about different skyscrapers at <http://www.pbs.org/wgbh/buildingbig/skyscraper/index.html>.
  - a) The Eiffel Tower is 986 feet tall. How far would you expect to see on a clear day?
  - b) The Empire State Building is 1250 feet tall. How far would you expect to see on a clear day?
  - c) The Petronas Towers in Malaysia are 1483 feet tall. How far would you expect to see on a clear day?

- d) Pollution cuts visibility. Assume that visibility cuts the distance that can be seen to 35% of the distance seen on a clear day. Write a function that describes distance on a cloudy day ( $C$ ) when given the distance that can be seen on a clear day ( $D$ ).
- e) Determine the number miles that can be seen from the three skyscrapers listed in parts a, b, and c.
- f) Write a function that describes the distance that can be seen on a cloudy day as a function of the height of the building. Explain the composition of functions.

- From the Eiffel Tower, one can see 38.5 miles. From the Empire State Building, one can see 43.1 miles, and from the Petronas Towers in Malaysia, one can see 47 miles.
- The function for cloudy distance is  $C(D) = 0.35D$ .
- If the day is cloudy the distances change to 13 miles for the Eiffel Tower, 15 miles for Empire State Building, and 16.5 miles for the Petronas Towers.
- For the cloudy distance based on the height of the building, the function is  $C(D(h)) = 0.35(1.22\sqrt{h})$  which is a composition.

2.

speed of car (mph)	0	22	33	44	55
braking distance (ft)	0	19	43	79	128

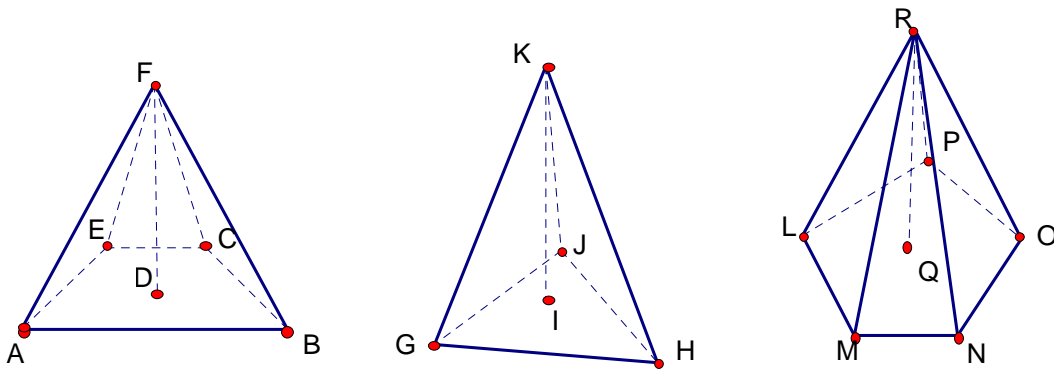
- a) Develop a mathematical model that allows us to predict braking distance given the speed of the car. (This data is extracted from a graph found on page 288 of the Glencoe Algebra 2 textbook.)
- b) Usually when there is a wreck police investigators are attempting to determine the speed of the driver. To do this, the investigators measure the length of the skid marks left on the pavement. This means the ordered pairs will be (braking distance, speed). How could we use the model from part a to create another function that will allow you to predict the speed of the driver given the length of the skid marks?  
[Hint: Simplify the model found in part a to include only the quadratic term assuming the other two terms (linear and constant) are so small that they do not affect the final outcome.]

Look at the data for braking distance as a function of speed by graphing L1, L2. This is clearly quadratic and can be fit with quadratic regression to produce  $l = 0.046s^2 - 0.199s + 0.264$ .

If we take our equation  $l = 0.046s^2 - 0.199s + 0.264$  and drop the constant and linear terms and simplify the coefficient of the quadratic term, we would have the model:  $l = 0.04s^2$ . If we solve for  $s$ , we get the equation  $s = 2\sqrt{5l}$ .

Student Handout  
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