

Applications of Rational Functions

Algebra 2

Goals:

1. Describe graphically, algebraically, and verbally real-world phenomena as functions; identify the independent and dependent variables. (3.01)
2. Use rational functions to solve problems. Solve by graphing, factoring, finding zeros and asymptotes through analysis of the polynomials in the numerator and denominator, and use properties of equality. (3.10)

Materials Needed:

1. Student Handout
2. Graphing calculator

Activity One: The Refrigerator: How much does it really cost?

On the web site http://www.eren.doe.gov/consumerinfo/energy_savers/appliancesbody.html there is a list of yearly cost for electricity for common household appliances.

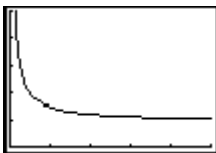
Appliance	Average Cost/year in electricity
Home Computer	\$9
Television	\$13
Microwave	\$13
Dishwasher	\$51
Clothes Dryer	\$75
Washing Machine	\$79
Refrigerator	\$92

- a) If we assume that a new refrigerator costs \$550, determine the total annual cost for a refrigerator that lasts for 15 years. Assume the only costs associated with the refrigerator are its purchase cost and electricity.
- b) Develop a function that gives the annual cost of a refrigerator as a function of the number of years you own the refrigerator.
- c) Sketch a graph of that function. What is an appropriate window?
- d) Since this is a rational function, determine the asymptotes of this function?
- e) Explain the meaning of the horizontal asymptote in terms of the refrigerator.
- f) If a company offers a refrigerator that costs \$1200, but says that it will last at least twenty years, is the refrigerator worth the difference in cost?

1.
$$\frac{550 + 92 \cdot 15}{15} = 128.67$$

2.
$$C(n) = \frac{550 + 92n}{n}$$
, where n = number of years and C = represents the annual cost.

3. The graph of the function is shown below with the window used.



```

WINDOW
Xmin=0
Xmax=50
Xscl=10
Ymin=0
Ymax=500
Yscl=100
Xres=1

```

4. The asymptotes of the rational function are $n = 0$ and $C = 92$. The n -value asymptote can be seen from the domain; you cannot substitute $n = 0$ in the function. The asymptote associated with C is more of a challenge. Students may explore values by using a table of values or by inspecting the function in the form

$$C(n) = \frac{550}{n} + 92. \text{ As } n \rightarrow \infty, \text{ the value of } C \text{ approaches } 92. \text{ From a practical view,}$$

the cost of the refrigerator goes to zero over many many years; however, the yearly expense of electricity continues.

5. The yearly expense of electricity continues no matter how many years the refrigerator works. The cost will never go below the \$92, but the cost approaches \$92.

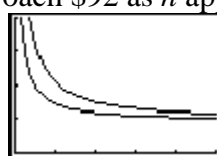
6. Graphing the two functions $C(n) = \frac{550+92n}{n}$ and $C_2(n) = \frac{1200+92n}{n}$ together or reviewing a table of values will show the more expensive refrigerator remains more expensive annually although both approach \$92 as n approaches infinity.

X	V ₂	V ₃
10	147	212
20	119.5	152
30	110.33	132
40	105.75	122
50	103	112
60	101.17	112
70	99.857	109.14

```

WINDOW
Xmin=0
Xmax=50
Xscl=10
Ymin=0
Ymax=400
Yscl=100
Xres=1

```

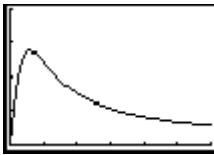


Activity Two: Drug Concentration

The function $C(t) = \frac{5t}{0.01t^2 + 3.3}$ describes the concentration of a drug in the blood stream over time. In this case, the medication was taken orally. C is measured in micrograms per milliliter and t is measured in minutes.

- Sketch a graph of the function over the first two hours after the dose is given. Label axes.
- Determine when the maximum amount of the drug is in the body and the amount at that time.
- Explain within the context of the problem the shape of the graph between taking the medication orally ($t = 0$) and the maximum point. What does the shape of the graph communicate between the maximum point and two hours after taking the drug?
- What are the asymptotes of the rational function $C(t) = \frac{5t}{0.01t^2 + 3.3}$? What is the meaning of the asymptotes within the context of the problem?
- Expand the window of the graph to include negative values for t . Discuss the asymptotes.

- Using the graphing calculator, students can produce a graph similar to the one shown.



WINDOW

Xmin=0
Xmax=180
Xsc1=30
Ymin=0
Ymax=20
Ysc1=5
Xres=1

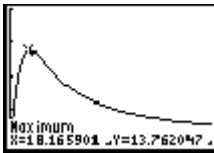
X	Y1
0	0
30	12.195
60	7.6336
90	5.3381
120	4.0733
150	3.2852
180	2.7498

X=0

where x represents time in minutes and y represents concentration in micrograms per milliliter.

Investigate the window settings with a table (shown here by increments of 30 minutes). Note that this window covers 3 hours and shows the concentration of the drug in the body over time.

- The maximum concentration in the body is 13.76 micrograms which occurs at 18.2 minutes.



- The graph shows that the concentration is 0 at time = 0. Within the first 15 minutes, the concentration rises sharply to the maximum at 18.2 minutes. After reaching the maximum, the kidneys begin cleansing the blood and rapidly remove the drug. The table show the concentrations at several times.

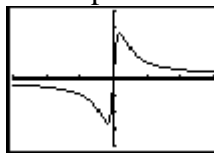
X	Y1
0	0
30	12.195
60	7.6336
90	5.3381
120	4.0733
150	3.2852
180	2.7498

X=0

- There is only one asymptote in this rational function which is at $C = 0$. Oddly, this value actually exists in the range of the function at $C = 0$; however, the graph tends toward $C = 0$ as t -values increase to infinity. This characteristic gives the asymptotic behavior. Graphs and tables of values help students see this.

WINDOW

Xmin=-300
Xmax=300
Xsc1=100
Ymin=-20
Ymax=20
Ysc1=5
Xres=1



X	Y1
-1500	-.3333
-1000	-.4998
-500	-.9987
0	0
500	.9986
1000	.4994
1500	.3332

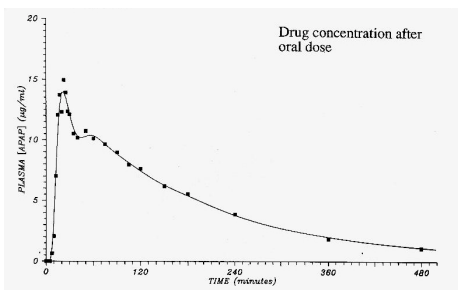
X=-1500

Also, comparing the degree of the numerator and the denominator identifies the “power” of the function is located in the denominator which means for larger and larger x -values, the y -values goes to 0.

The meaning of the asymptote is associated with the concentration of the drug in the bloodstream approaches 0 over time. (Thank goodness!)

- Use graphs shown in the answer to question 4 above.

The web site http://www.maxent.co.uk/images/drugs_2.jpg contains graphs that show drug concentrations in the body over time.



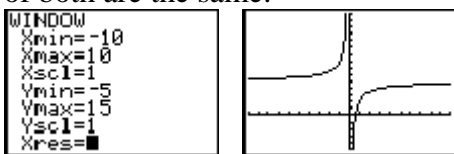
Follow-Up Problems:

1. Tiffany and Adam are graphing functions at the chalk board. Tiffany is graphing

$$f(x) = \frac{5x-3}{x} \text{ and Adam is graphing } g(x) = 5 - \frac{3}{x}.$$

- The teacher asks both students to first write the domain of each of the functions. What should Tiffany say? What should Adam say?
- Because these are both rational functions, the teacher asks both students to identify the asymptotes for each of the functions. What should each person say?
- Sketch the graphs of each function and check on the graphing calculator.
- Surprised? Write an explanation of why these functions produce the same graph and then point out the easy information that comes from each form.

This problem is designed to help students see that algebra can help them. Both Tiffany and Adam are graphing the same function. The domains for both are $x \neq 0$. The asymptotes for both are $x = 0$ and $y = 5$. The form of the function that Adam is graphing shows more clearly that the y -value will never be 5. The sketches for graphs of both are the same:



Rewriting function f as function g makes the horizontal asymptote much easier to identify.

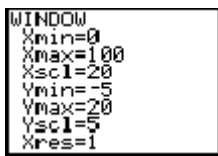
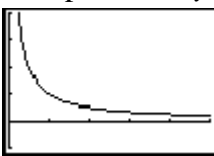
- Your family is traveling in your car for 100 miles.
 - How long will the trip take if you average 30 miles per hour, 55 miles per hour, or 65 miles per hour?
 - Write a function that describes the time it takes to make this trip as a function of your speed. Identify the meaning of the variables.
 - Graph this function. Show asymptotes.

What does this graph tell you about the time it will take you to travel depending on the speed of the car?

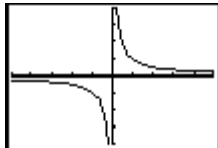
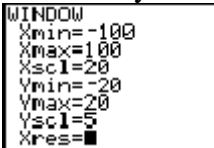
The calculations for values to answer the first question identifies a procedure:

$\frac{\text{distance}}{\text{rate}} = \text{time}$ which comes from $\text{rate} \cdot \text{time} = \text{distance}$. For 30 mph, the time is 3.33 hours; for 55 mph, 1.82 hours; and 65 mph, 1.53 hours. The faster you travel, the fewer

hours you are on the road, The function that describes this relationship is $y = \frac{100}{x}$, where $x = \text{speed}$ and $y = \text{time}$.



The asymptotes are $y = 0$ and $x = 0$. With the window expanded, the asymptotes are more easily identified:



3. For what positive number is the sum of the number and its reciprocal smallest?

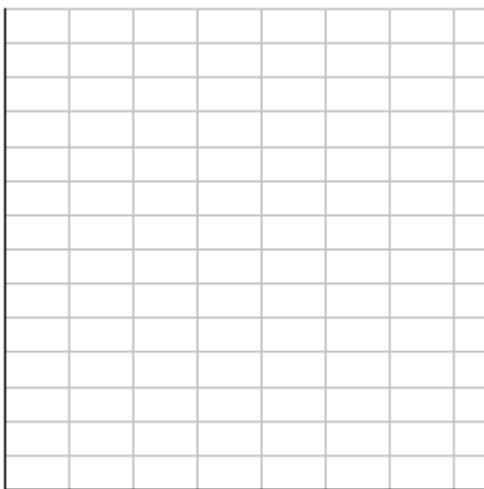
If x is the number, then $\frac{1}{x}$ is the reciprocal. Therefore the sum of a number and its reciprocal is represented by $x + \frac{1}{x}$. Graph the function $y = x + \frac{1}{x}$ for positive x -values. The minimum occurs at the point (1,2).

Student Handout
Applications of Rational Functions
Algebra 2

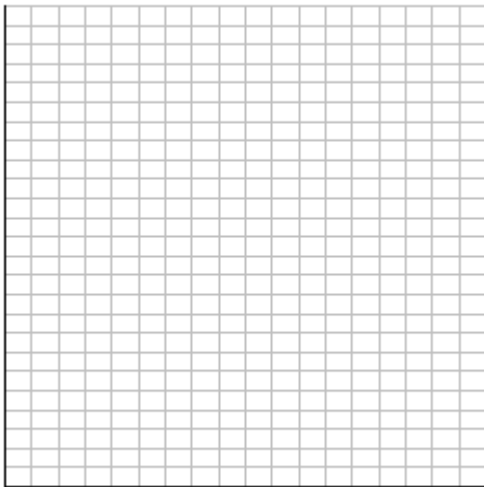
1. On the web site http://www.eren.doe.gov/consumerinfo/energy_savers/appliancesbody.html there is a list of yearly cost for electricity for common household appliances.

Appliance	Average Cost/year in electricity
Home Computer	\$9
Television	\$13
Microwave	\$13
Dishwasher	\$51
Clothes Dryer	\$75
Washing Machine	\$79
Refrigerator	\$92

- If we assume that a new refrigerator costs \$550, determine the total annual cost for a refrigerator that lasts for 15 years. Assume costs include purchase and electricity.
- Develop a function that gives the annual cost of a refrigerator as a function of the number of years you own the refrigerator.
- Sketch a graph of that function. What is an appropriate window? Label scale.
- Since this is a rational function, determine the asymptotes of this function?
- Explain the meaning of the horizontal asymptote in terms of the refrigerator.
- If a company offers a refrigerator that costs \$1200, but says that it will last at least twenty years, is the refrigerator worth the difference in cost?



2. The function $C(t) = \frac{5t}{0.01t^2 + 3.3}$ describes the concentration of a drug in the blood stream over time. In this case, the medication was taken orally. C is measured in micrograms per milliliter and t is measured in minutes.
- Sketch a graph of the function over the first two hours after the dose is given. Label axes.
 - Determine when the maximum amount of the drug is in the body and the amount at that time.
 - Explain within the context of the problem the shape of the graph between taking the medication orally ($t = 0$) and the maximum point. What does the shape of the graph communicate between the maximum point and two hours after taking the drug?
 - What are the asymptotes of the rational function $C(t) = \frac{5t}{0.01t^2 + 3.3}$? What is the meaning of the asymptotes within the context of the problem?
 - Expand the window of the graph to include negative values for t . Discuss the asymptotes.



Follow-Up Problems
Applications of Rational Functions
Algebra 2

1. Tiffany and Adam are graphing functions at the chalk board. Tiffany is graphing $f(x) = \frac{5x-3}{x}$ and Adam is graphing $g(x) = 5 - \frac{3}{x}$.
 - a. The teacher asks both students to first write the domain of each of the functions. What should Tiffany say? What should Adam say?
 - b) Because these are both rational functions, the teacher asks both students to identify the asymptotes for each of the functions. What should each person say?
 - c) Sketch the graphs of each function and check on the graphing calculator.
 - d) Surprised? Write an explanation of why these functions produce the same graph and then point out the easy information that comes from each form.

2. Your family is traveling in your car for 100 miles.
 - a) How long will the trip take if you average 30 miles per hour, 55 miles per hour, or 65 miles per hour?
 - b) Write a function that describes the time it takes to make this trip as a function of your speed.
 - c) Graph this function. Show asymptotes.
 - d) What does this graph tell you about the time it will take you to travel depending on the speed of the car?

3. For what positive number is the sum of the number and its reciprocal smallest?