The Distance Formula
Algebra 2

Goals:

1. Describe graphically, algebraically and verbally real-world phenomena as functions; identify independent and dependent variables. (3.01)
2. Translate among graphic, algebraic, and verbal representations of relations. (3.02)
3. Use equations that contain radical expressions to solve problems. Solve by (3.11)
    a. Graphing
    b. Factoring
    c. Using properties of equality; justify steps used.

Materials and Equipment Needed:

1. Copy of Student Handout for each student
2. Ruler that shows centimeters
3. Blank unlined paper for each student
4. Calculator

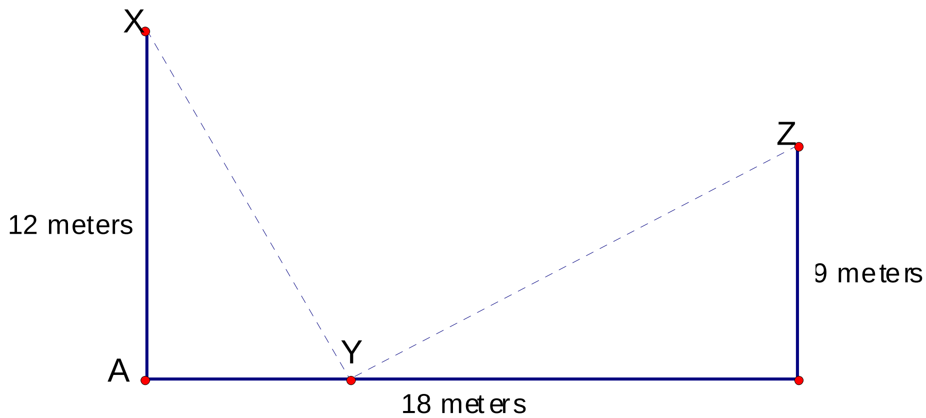
Activity 1: Adding two distances

The algebra classes are having a competition that involves brain and brawn. A student is given a problem statement at point X, runs to a point Y on the table where this person solves the problem, and once finished with the problem races to point Z where answers are turned in. Both time and correct answers are considered in declaring a winner.

The brain factor of the competition will rely on our mathematical knowledge and quick thinking. However, to be on the safe side our team wants to find the shortest distance to minimize the running time for the race. What point Y creates the shortest distance?

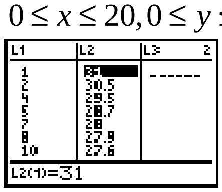
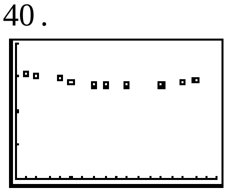
Point X is 12 meters from the table, the table is 18 meters long, and point Z is 9 meters from the table.

a) Complete a scaled drawing to investigate. Use centimeters instead of meters for the drawing. (Drawing below is not scaled.)

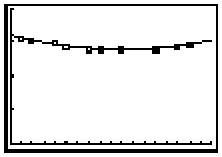


b) There are many possible locations for point Y. You and your partner find the total length of $XY + YZ$ for several different points Y. To keep up with the location of point Y, measure the length of AY. Fill in table with values from different locations.
c) What is the best location for point Y? Remember we want to keep the distance as short as possible.
d) Make a scatter plot of the data with values of AY in list 1 and values of the sum $XY + YZ$ in list 2.
e) Develop a function that describes the distance $XY + YZ$ where we know the length $AY = x$.
f) Graph that function.
g) Find the shortest distance using the function.
h) Go back to your scale drawing to locate the point Y that creates the shortest distance.

1. Separate the group into partners.
2. Using rulers create the scale drawing (one on handout is not drawn to scale) and find the measures specified. Fill in the table of values. Suggest that measurements be made to the nearest tenth of centimeter.
3. Have students enter data into lists in the calculator—putting lengths of AY in L1 and values of XY+YZ in L2 and create a scatter plot. A suggested window is $0 \le x \le 20, 0 \le y \le 40$.

4. You might suggest that students put a coordinate system over the scale drawing—perhaps placing (0,0) at point M. Develop a function to describe the distances: $D(x) = \sqrt{x^2 + 144} + \sqrt{(18 - x)^2 + 81}$. When we plot this function, it should pass close to the points of the scatter plot.



5. Find the shortest distance using the Minimum option from 2nd Trace or CALC menu. This minimum value occurs at (10.28, 27.66).
6. Go back to the scale drawing to locate the A that creates the shortest distance.

Activity 2: Distance between a point and a curve.

One of the members of Survivor is wandering in the desert along the path $y = 0.5x^2 + 1$. There is a water fountain located at the point $(5, -3)$. At what point of the path is the member closest to the water? How far away is she from the fountain?

1. First, take a look at the situation using the graphing calculator. Graph the function and plot the point by putting the value 5 in list 1 and –3 in list 2 and turning on a stat plot. A standard window seems to illustrate the path and the point.
2. From the illustration, we can estimate when the member of Survivor is closest to the water fountain. To be sure we will need some mathematical procedure.
3. Using the distance formula, we can find the distance between the points (x, y) on the curve and $(5, -3)$. $D = \sqrt{(x-5)^2 + (y+3)^2}$. This function must be adjusted since there are three variables. Let $y = 0.5x^2 + 1$. The function to describe distance is transformed to $D = \sqrt{(x-5)^2 + (0.5x^2 + 1 + 3)^2}$.
4. A graph of this new function shows the distances. We want the smallest distance. Using a window of $-5 \le x \le 5; 0 \le y \le 10$, we can see a point that is the minimum. Use 2nd Trace or the CALC menu to find the minimum. The minimum found by the calculator is $(0.92, 6.02)$. Which means the survivor is closest to the water fountain when the $x-$value is 0.92. Go back to the original graph and give that value for *x* to see when she is closest.

Follow Up Problem:

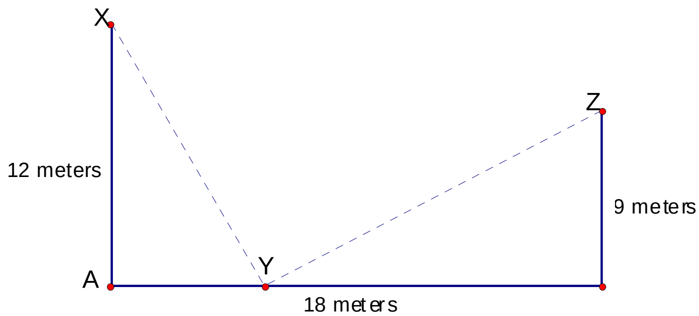
A graph of the post offices and the highway shows that the location of the new post office will be (*x*, 0). It is useful to have students do a good drawing (you may want a scale drawing like done in the water pail problem above). They must attach a coordinate system to this sketch. The function that will sum the distances of the new post office to each of the existing post offices is
$y = \sqrt{(x-0)^2 + (0-0)^2} + \sqrt{(x-3)^2 + (0-5)^2} + \sqrt{(x-6)^2 + (0+8)^2}$. Enter this function into the calculator and locate the minimum of the function. This exists at $(0.564, 15.8)$.

Student Handout
The Distance Formula

1. The algebra classes are having a competition that involves brain and brawn. A student is given a problem statement at point X, runs to a point Y on the table where this person solves the problem, and once finished with the problem races to point Z where answers are turned in. Both time and correct answers are considered in declaring a winner.

The brain factor of the competition will rely on our mathematical knowledge and quick thinking. However, to be on the safe side our team wants to find the shortest distance to minimize the running time for the race. What point Y creates the shortest distance?

Point X is 12 meters from the end of the table, the table is 18 meters long, and point Z is 9 meters from the other end of the table.



a) Complete a scaled drawing to investigate. Use centimeters instead of meters for the drawing. (Drawing below is not scaled.)

AY										
$XY+YZ$										

b) There are many possible locations for point Y. You and your partner find the total length of $XY+YZ$ for several different points Y. To keep up with the location of point Y, measure the length of AY. Fill in table with values from different locations.

c) What is the best location for point Y? Remember we want to keep the distance as short as possible.

d) Make a scatter plot of the data with values of AY in list 1 and values of the sum $XY+YZ$ in list 2.

e) Develop a function that describes the distance $XY+YZ$ where we know the length $AY=x$.

f) Graph that function.

g) Find the shortest distance using the function.

h) Go back to your scale drawing to locate the point Y that creates the shortest distance.

2. One of the members of Survivor is wandering in the desert along the path $y=0.5x^2+1$. There is a water fountain located at the point $(5,-3)$. At what point of the path is the member closest to the water? How far away is she from the fountain?

Follow Up Problem
The Distance Formula

The government is attempting to locate a new post office to replace three existing post offices. The goal is to locate the new post office to be closest to the existing post offices and to lie on a mail highway that runs through the region.

If a coordinate system is superimposed over a map of the region, the main highway is located along the x-axis and the three post offices are located at (0,0), (3,5), and (6, -8). Give the coordinates of the location of the new post office.