

## Questions about Quadratics

### Algebra 2

#### Goals:

1. Operate with algebraic expressions to solve problems. (1.02)
  - a) Factor polynomials and other algebraic expressions completely over the real numbers.
2. Graph relations and functions and find the zeros of the functions. (3.03)
3. Use quadratic equations and inequalities to solve problems. Solve by: (3.05)
  - a. Graphing.
  - b. Factoring.
4. Find and interpret the maximum and minimum values and the intercepts of a quadratic function (3.06)
5. Use equations which contain radical expressions to solve problems. Solve by: (3.11)
  - a. Graphing
  - c. Using properties of equality.

#### Materials and Equipment Needed

1. Copies of the handout for each student
2. Graphing calculator
3. Paper for note taking.

#### Activity One: The Calculator as an Aid in Solving Quadratic Equations.

The following problems are taken from the Algebra II Indicators and were developed by the North Carolina Department of Public Instruction.

Using the following problems, we will explore the use of the graphing calculator as an aid in completing the analytical tasks of quadratic equations: completing the square, factoring, and use of the quadratic formula. First, freely use the calculator to solve the problem. Then use analytical methods to solve the problems while using the calculator as a support in your work.

1. The function  $f(x) = -0.019x^2 + 3.04x - 58.87$  describes newspaper circulation (in millions) in the United States for 1920 to 1998 (where  $x = 20$  is used for 1920). Identify periods of increasing and decreasing circulation. According to the function, when did newspaper circulation peak? When will circulation approximate 45 million?
2. For  $y = 2x^2 - 17x + 26$ , locate exactly the vertex and  $x$ - and  $y$ -intercepts.

The focus for these problems is to use the calculator to answer the questions but to also use paper and pencil skills to answer the questions.

1. Graph the function  $f(x) = -0.019x^2 + 3.04x - 58.87$  on the calculator. The window should reflect the use of the function to describe newspaper circulation. Suggested:  $0 \leq x \leq 150, -20 \leq y \leq 100$ . Using 2<sup>nd</sup> Trace for the CALC menu, find the maximum point of the function (80, 62.73), and the zeros (137.5, 0) and (22.5, 0). Since this is a max, the increasing period goes from the first zero to the max or  $22.5 < x < 80$ , and the decreasing period goes from the max to the second zero  $80 < x < 137.5$ . The peak is

reached at the max (80, 62.73). Graph the line  $y = 45$  and find the point of intersection of the parabola and the line to determine when the circulation is 45 billion;  $y = 45$  when  $x = 49.5$  or  $110.5$ .

Now do this problem analytically by first writing the equation in vertex form by

completing the square.  $y = -0.019(x^2 - 160x + 6400) - 58.87 + 121.6$ . We get the same  
 $y = -0.019(x - 80)^2 + 62.73$

result as from the calculator. To find the zeros, we must solve

$$0 = -0.019(x - 80)^2 + 62.73$$

$$\frac{-62.73}{-0.019} = (x - 80)^2$$

$$\pm\sqrt{3301.578947} + 80 = x$$

or use of the quadratic formula  $\frac{-3.04 \pm \sqrt{(-3.04)^2 - (4 \cdot -.019 \cdot -58.87)}}{2 \cdot -.019}$ .

We get the same results with much more pain.

2. We can practice the same techniques using  $y = 2x^2 - 17x + 26$ . By completing the square we find the vertex, and using the quadratic formula or solving for zeros from the equation that results from completing the square, we find the  $x$ -intercepts. The  $y$ -intercept is found by letting  $x = 0$  in the equation.

$$y = 2\left(x^2 - \frac{17}{2}x + \frac{289}{16}\right) + 26 - \frac{289}{8} \quad \text{or} \quad x = \frac{17 \pm \sqrt{(-17)^2 - 4 \cdot 2 \cdot 26}}{2 \cdot 2}$$

$$y = 2\left(x - \frac{17}{4}\right)^2 - \frac{81}{8} \quad x = 2 \text{ or } 6.5$$

The  $y$ -intercept is (0, 26). In the quadratic formula, can show the kids the use of the alpha keys for storage of values and then input the quadratic formula with alpha keys. Now do this problem on the calculator.

The goal of this exercise is to show the ease of use of the calculator and how the analytical can be verified with the calculator.

### Follow-Up Activity

Given the function  $f(x) = 2.5x^2 + 15x + c$ , where  $c$  is a real number. For different values of  $c$ , the graph has different characteristics.

- a. Give several examples of values of  $c$  and the characteristics of the graph.
  - b. Determine values for  $c$ -values that produce common characteristics.  
 (This question is adapted from the Indicators for Algebra II prepared by DPI.)
1. By selecting some values of  $c$  and looking at the graph, the value of  $c$  determines the  $y$ -intercept of the function. Also for some  $c$ -values there are two points of intersection with the  $x$ -axis, and sometimes only one intersection point, and sometimes no point of intersection.

For example, if  $c = 0$ , there are two points of intersection and  $(0,0)$  is the  $y$ -intercept. If  $c = 22.5$ , there is only one point of intersection and  $22.5$  is the  $y$ -intercept. If  $c = 30$ , the curve does not cross the  $x$ -axis and the  $y$ -intercept is  $30$ .

2. By rewriting the function using the method called completing the square, the boundaries “show” themselves.

$$\begin{aligned}y &= 2.5x^2 + 15x + c \\ &= 2.5(x^2 + 6x + \quad) + c \\ &= 2.5(x^2 + 6x + 9) + c - 22.5 \\ &= 2.5(x + 3)^2 + c - 22.5\end{aligned}$$

This form shows us that if we solved for zeros by letting  $y = 0$ , the equation transforms to  $22.5 - c = 2.5(x + 3)^2$ . If  $c = 22.5$ , there is one root. If  $c > 22.5$ , the perfect square would equal a negative number, and there is no real root. If  $c < 22.5$ , the perfect square is equal to a positive number and two roots are produced.

A similar analysis can be made based on the quadratic formula and using only the discriminant  $b^2 - 4ac$  to determine values of  $c$  that make this expression positive, zero, or negative.

Student Handout  
Questions about Quadratics  
Algebra 2

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