Algebra 2
Pig Problem: Writing and Solving Quadratic Equations

Goals:

1. Describe graphically, algebraically, and verbally real-world phenomena as functions: identify the independent and dependent variables (3.01)
2. Use quadratic equations and inequalities to solve problems. Solve by (3.04)
   a. graphing
3. Find and interpret the maximum and minimum values and the intercepts of a quadratic function. (3.06)

Although this goal is not mentioned in the NC Mathematics Curriculum, we will translate verbal situations into mathematical statements or expressions.

Materials Needed:

1. paper for note taking
2. pencil
3. calculator
4. copy of student handout

Activity One: Selling the pig for the best price.
There is an animation to describe the issues of the pig gaining weight and decreasing in worth.

1. Pink Pig Problem

Farmer Fay has a pig that presently weighs 200 pounds. She could sell it now for a price of $1.40 a pound. The pig is gaining 10 pounds a week while the price per pound of pork is dropping 2 cents a week.

a) When should Fay sell the pig to get the maximum amount of money for it?
b) If Fay pays $5 per week for food for the pig, when should Fay sell the pig to get the maximum profit?

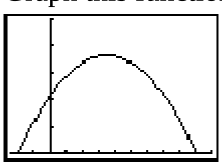
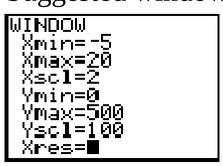
1. Discuss the problem setting with the students and look at the animation which illustrates the problem. Be sure the word "worth" is understood.
2. Begin by finding some values of worth. for now? next week? week after next? Analyze the method used to find these values. Begin to develop a formula. Answers: $\text{now} = 200(\$1.40) = \$280.00$, $\text{next week} = 210(\$1.38) = \$289.80$, week after $\text{next} = 220(\$1.36) = \$299.20$.
3. What is the independent variable of this problem?
4. Write a function to describe the worth of the pig. $f(t) = (200 + 10t)(1.40 - .02t)$ where t is the number of weeks.
5. The maximum amount of money is found best from the graph, although it can be done from the table as well. For a graph, discuss the window: $0 \le t \le 50, 0 \le f(t) \le 500$. The maximum of the graph can be found at (25, 405).
6. If the window is expanded to $0 \le t \le 100, 0 \le f(t) \le 500$, students can see that the pig will be worth nothing 70 weeks from now. This question invites students to either find a zero using the calculator or to solve the equation: $(200 + 10t)(1.40 - .02t) = 0$. This can be done by setting the factors equal to 0 and solving the linear equation.
7. The issue of feeding the pig changes the function. If the pig costs $5 each week to feed, this reduces profit. The new function to describe the worth of the pig is $W(t) = (200 + 10t)(1.40 - .02t) - 5t$. Using the window of $0 \le t \le 50, 0 \le f(t) \le 500$, the

graph has a new maximum: (12.5, 311.25). If we must hold to an integer week number, then either week 12 or week 13 would be the best time to see the pig (12.5 is not an integer). At this time, the pig would bring $311.20.

Activity Two: Zena and Radar Problem
There is an animation for this problem to describe the size and shape of the room over time.

2. Zena and Radar crash landed on Planet X. Their captors have locked them in a space that is 18 feet long and 12 feet wide. But the length is decreasing linearly with time at a rate of 1 foot per minute, and the width is increasing linearly with time at a rate of 3 feet per minute. When will the area of the room reach a maximum value? When will the area of the room reach a minimum value?

1. Discuss the problem setting with the students and view the animation that illustrates the problem.
2. Talk about the meaning of specific words: "decreasing linearly" and "increasing linearly". What is the independent variable of the problem?
3. Talk about simplifying the problem as was illustrated in the previous problem:
    a. Find specific values of area: what is the area now? in the next minute? in two minutes? $\text{now} = 18 \times 12 = 216$ square meters, in one $\text{minute} = (18-1)(12+3) = 255$, in two $\text{minutes} = (20-2)(12+6) = 288$.
    b. Analyze the method used to find these values—working to identify algebraic expressions.
    c. find length and width as a function of *t*. $\text{length} = 18 - t$ and $\text{width} = 12 + 3t$.
4. The produce of length and width produces a function that describes the area as a function of time. $A(t) = (18-t)(12+3t)$.
5. Graph this function. Suggested window: $-5 \le t \le 20; 0 \le A(t) \le 500$.

6. Find a maximum using the calculator: (7, 363).
7. Find a minimum value—need to talk about the real life aspects of the problem: this question is asking for the zero. Again either using the calculator of by solving the equation: $(18-t)(12+3t) = 0$ we find the zero at $t = 18$. Again, the value $t = -4$ does not fit this real context.

Follow Up Problem: Small company makes monitors.
A small electronics company builds special large monitors for use in newspaper layout. Currently the company employees 5 workers who each make on average 30 monitors per day. Demand for these monitors is rising and the company wants to increase production. However, since the building space is small and tools are in limited supply, production per worker decreases by 2 monitors per day with every new employee hired. What total number of workers allows the company to manufacture the largest number of monitors each day? How many monitors will be produced each day with this work force? If too many workers are hired there is so much confusion that no monitors are built in a day. How many workers makes this happen?

1. There are many approaches to this problem. Students could simply make lists of numbers of workers and the number of monitors built per worker each day, find the product, and determine the maximum.

2. If the function approach is used, the resulting function is $w(x) = (30 - 2x)(5 + x)$, where $x =$ new workers.
3. A graph of this function produces a quadratic shape with a maximum of (5, 200) where 5 represents the number of new workers to be hired and 200 is the resulting number of monitors produced each day. The total number of workers will then be 10.
4. The chaos that makes no monitors be built occurs when $y = 0$. Using the calculator to find the zero, we discover the point (15,0) or the company has a total of 20 workers.

Student Handout
Algebra 2
Writing Quadratic Functions and Solving Quadratic Equations

1. Pink Pig Problem
Farmer Fay has a pig that presently weighs 200 pounds. She could sell it now for a price of $1.40 a pound. The pig is gaining 10 pounds a week while the price per pound of pork is dropping 2 cents a week.

c) When should Fay sell the pig to get the maximum amount of money for it?
d) If Fay pays $5 per week for food for the pig, when should Fay sell the pig to get the maximum profit?

2. Zena and Radar crash landed on Planet X. Their captors have locked them in a space that is 18 feet long and 12 feet wide. But the length is decreasing linearly with time at a rate of 1 foot per minute, and the width is increasing linearly with time at a rate of 3 feet per minute. When will the area of the room reach a maximum value? When will the area of the room reach a minimum value?

Follow Up
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