

## Algebra 2

### Linear Programming

#### Goals:

1. Describe graphically, algebraically, and verbally real-world phenomena as functions; identify the independent and dependent variables (3.01)
2. Translate among graphic, algebraic, and verbal representations of relations. (3.02)
3. Graph relations and functions. (3.03)
4. Use systems of two or more equations to solve problems. Solve by graphing. (3.12)
5. Use linear programming to solve problems. (3.13)

#### Materials and Equipment Needed:

1. Graph paper
2. Copy of handout for each student
3. Graphing calculator

#### Activity One: Getting started with linear programming. This problem is based on a problem in the Algebra II Indicators for Goal 1.13

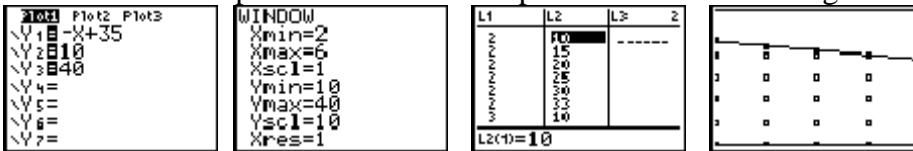
During the summer break, Allie works as many as 35 hours per week. On Saturdays she spends between two and six hours teaching adults to sail. On weekdays she can work between 10 and 40 hours teaching sailing at a children's camp. Teaching adults pays \$10 per hour while teaching children pays \$6.25 per hour. How can Allie earn the most money each week?

1. We will begin the discussion with the naming of all possible variables in this problem. These might include: **total time, time working with adults, time working with children, amount earned**. In this discussion we would realize that the value of **time working with adults (A)** and **time working with children (C)** determines the values of all the variables.
2. Using different sentences and phrases from the problem, we can write several algebraic statements:
  - a.  $2 \leq A \leq 6$  describes possible time for working with adults
  - b.  $10 \leq C \leq 40$  describes possible time for working with children
  - c.  $C + A \leq 35$  describes the possible total time for work
  - d.  $\text{Salary} = 10A + 6.25C$  describes the total salary for the week.
3. Next we begin a graphical illustration of these statements. The salary statement we will consider last. Using graph paper students decide which variables will be plotted as dependent and as independent variables (for uniformity, we will use A on the horizontal axis and C on the vertical axis). From this decision, we need then to establish a scale for the graphs. On graph paper, they will sketch and shade regions for the following three inequalities:  $2 \leq A \leq 6$ ,  $10 \leq C \leq 40$ ,  $C + A \leq 35$ .
4. Using a **powerpoint** prepared by the Distance Learning department, we will be able to see the different regions overlapping.

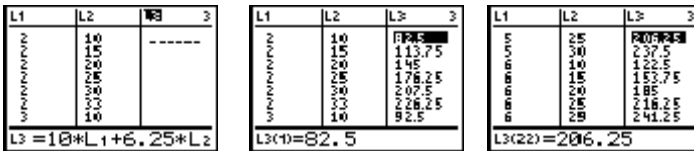
- Determine possible ordered pairs for the number of hours Allie can work with adults (A) and the number of hours she can work with children (C). For simplification, we will count by fives for time worked with children except near the top boundary. Possible ordered pairs are:

$$\left\{ \begin{array}{l} (2,10), (2,15), (2,20), (2,25), (2,30), (2,33), (3,10), (3,15), (3,20), (3,25), (3,30), (3,32), \\ (4,10), (4,15), (4,20), (4,25), (4,30), (4,31), (5,10), (5,15), (5,20), (5,25), (5,30), (6,10), \\ (6,15), (6,20), (6,25), (6,29) \end{array} \right\}$$

- To consider this on the graphing calculator, plot  $y_1 = -x + 35$ ,  $y_2 = 10$ , and  $y_3 = 40$ . The constraints on variable A will have to be seen through limitation of the window. A suggested window is  $2 \leq x \leq 6, 10 \leq y \leq 40$ .
- Using lists, enter the possible ordered pairs from 5 above to plot as a scatter plot. Use L1 for Adult times and L2 for corresponding Children times. Once this is done the window can be expanded because these points will define the region.



- Now for the question of determining the point or points that create maximum salary. Remember that  $\text{Salary} = 10A + 6.25C$ . Use the lists to find the values of  $10A + 6.25C$  where values of A are in L1 and values of C are in L2. Create these values in L3 by inputting at the top of L3:  $L3 = 10 * L1 + 6.25 * L2$ . Now search L3 for maximum values.



- The maximum value for salary occurs at the ordered pair (6,29). This maximum value is \$241.25. For the data, we counted by fives. Does that make a difference in this answer?
- Next we want to create a uniform way to solve problems like these. Hopefully, we will draw from this discussion the need to establish the region on the graph that fulfills the description and then see that the only reasonable answers occur at the vertices—whether we are searching for a maximum or a minimum. Therefore, it is not necessary to identify every possible solution, but just check the vertices of the region. In general mathematicians have proved that the maximum and minimum values will occur at the boundary points of our feasible region.

**NOTE:** Our goal is to find the maximum value of  $10A + 6.25C$  within in this feasible region. Substituting each of the boundary points in the expression  $10B + 30M$  shows that the point (6,29) gives the largest value for profit.

To think of this graphically, the expression for the profit,  $10A + 6.25C$ , has different values for different values of A and C. Let  $P = 10A + 6.25C$ . Using A as the independent variable and C as the dependent variable, solving for C in terms of A and P, the equation

$P = 10A + 6.25C$  can be rewritten as  $C = \frac{-10}{6.25}B + \frac{P}{6.25}$ . Now if we pick specific values for  $P$ ,  $C = \frac{-10}{6.25}B + \frac{P}{6.25}$  forms a group of linear equations whose graphs are parallel lines. The graph below shows some of these lines superimposed over the constraint lines.



As we look at the parallel lines that fall within the feasible region, we can see that the profit will reach its maximum value at the boundary point (6,29). Therefore, Allie's maximum profit will be  $10 \cdot 6 + 6.25 \cdot 29 = 241.25$  or \$241.25.

### Follow-Up Activity - The Pottery Problem

A potter is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses 0.75 lb of clay and each plate uses 1 lb of clay. She has 20 hours available for making the cups and plates and has 250 lbs of clay on hand. She makes a profit of \$2 on each cup and \$1.50 on each plate. How many cups and how many plates should she make to maximize her profit? (From *The NCSU Finite Math Book*, L.B. Page and S.O. Paur, 1991).

#### Solution:

1. The variables are **C** for the number of cups and **P** for the number of plates.
2. Converting time in minutes to time in hours we have,  $\frac{6}{10}C + \frac{3}{60}P \leq 20$  which describes the number of hours she will spend on this work. Re-writing the inequality, we have  $0.10C + 0.05P \leq 20$ .
3.  $0.75C + P \leq 250$  describes the amount of clay to be used.
4. The profit is described by the sum  $2C + 1.50P$ . For given ordered pairs  $(C, P)$  that meet the requirements of the two inequalities above, we want the ordered pair that gives the maximum value to  $2C + 1.50P$ .
5. Either  $C$  or  $P$  can be the independent variable. In this case, we will use  $C$  as the independent variable. Transforming the two inequalities to show  $P$  as a function of  $C$  gives:  $P \leq -2C + 400$  and  $P \leq -0.75C + 250$ . Graphs of these inequalities are shown with the point of intersection of the two lines. The boundary points of the feasible region of the plane are:  $(0, 250)$ ,  $(120, 160)$ ,  $(200, 0)$ , and  $(0, 0)$ . The following graph shows the feasible region.



6. Our goal is to find the maximum value of  $2C + 1.50P$  within in this feasible region. Substituting each of these boundary points in the expression  $2C + 2.50P$  shows that the point  $(120,160)$  gives the largest value for profit. That value is \$480.
7. To think of this graphically, the expression for the profit,  $(120,160)$ , has different values for different values of  $C$  and  $P$ . Let  $R = 2C + 1.50P$ , where  $R$  can range through different values. Using  $C$  as the independent variable and  $P$  as the dependent variable, solving for  $P$  in terms of  $R$ , the equation  $R = 2C + 1.50P$  can be rewritten as

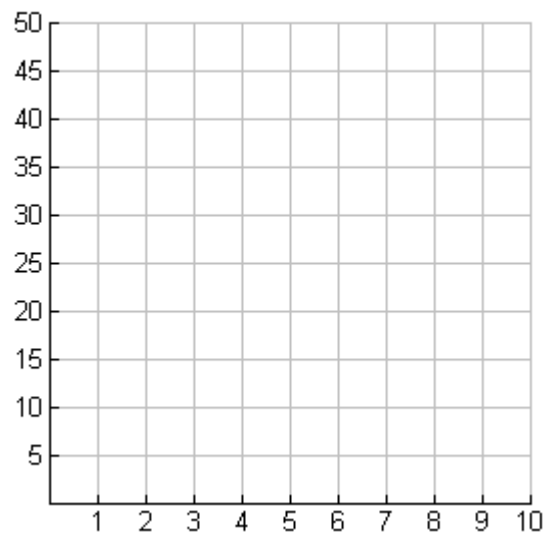
$$P = \frac{-2}{1.50}C + \frac{R}{1.50}. \text{ If we pick specific values for } R,$$

8.  $P = \frac{-2}{1.50}C + \frac{R}{1.50}$  forms a group of parallel linear equations. The graph below shows some of these lines superimposed over the constraint lines associated with  $0.10C + 0.05P \leq 20$  and  $0.75C + P \leq 250$ . As we look at the parallel lines that fall within the feasible region, we see that the profit will reach its maximum value at the corner point  $(120,160)$ . Therefore, the potter's maximum profit will be  $\$2 \cdot 120 + \$1.50 \cdot 160 = \$480$ . Please take a minute to run the **animation** from the website. As we see the lines "flow" across the feasible region we're looking for the line that represents the largest profit value and still remains in the feasible region.



Student Handout  
 Linear Programming  
 Algebra 2

1. During the summer break, Allie works as many as 35 hours per week. On Saturdays she spends between two and six hours teaching adults to sail. On weekdays she can work between 10 and 40 hours teaching sailing at a children's camp. Teaching adults pays \$10 per hour while teaching children pays \$6.25 per hour. How can Allie earn the most money each week?
  - a. What are the variables in this question?
  - b. Write algebraic statements that describe the information given in the problem description.
  - c. Sketch a graph of these algebraic statements.



- d. List all the ordered pairs that are possible solutions.
- e. Determine which of these ordered pairs produce the largest salary.

Follow-Up Problem - The Pottery Problem  
Linear Programming  
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A potter is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses 0.75 lb of clay and each plate uses 1 lb of clay. She has 20 hours available for making the cups and plates and has 250 lbs of clay on hand. She makes a profit of \$2 on each cup and \$1.50 on each plate. How many cups and how many plates should she make to maximize her profit? (From *The NCSU Finite Math Book*, L.B. Page and S.O. Paur, 1991).